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AFAPL-TR-65-45
Part VI

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY
Part VI: The Influence of Electromagnetic Forces on the
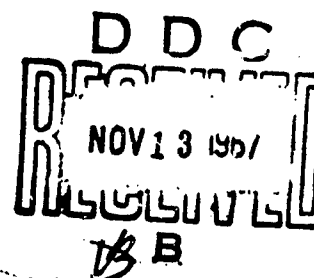Stability and Response of an Alternator Rotor.

J. Lund
T. Chiang

Mechanical Technology Incorporated

TECHNICAL REPORT AFAPL-TR-65-45, PART VI

September 1967

Air Force Aero Propulsion Laboratory
Directorate of Laboratories
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio

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AFAPL-TR-65-45
Part VI

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY

Part VI: The Influence of Electromagnetic Forces on the Stability and Response of an Alternator Rotor.

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ABSTRACT

This volume presents an analytical investigation of the vibrations induced in an alternator rotor by the generated electromagnetic forces. Formulas are given from which the magnetic forces can be calculated for three brushless alternator types: 1) the homopolar generator, 2) the heteropolar inductor generator, and 3) the two-coil Lundell generator. Numerical examples are given to illustrate the practical use of the formulas.

Two computer programs have been written for evaluation of the effect of the magnetic forces on the rotor. Manuals are provided for both programs, containing listings of the programs and giving detailed instructions for preparation of input data and for performing the calculations. The first computer program examines the stability of the rotor and the second program calculates the amplitude response of the rotor resulting from a built-in eccentricity and misalignment between the axes of the rotor and the alternator stator. Sample calculations are provided.

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SYMBOLS

A	Cross-sectional area of a shaft section, inch^2
A	Area of a pole, inch^2
A_N, A_S	Area of a north pole or a south pole, inch^2
A_T	Area of a stator tooth in the heteropolar generator, inch^2
A, B	Influence matrices for rotor, eq. (H.62), Appendix H
a	$= -\dfrac{AB_o^2}{72C} L_p$, magnetic force gradient for 4 pole homopolar generator, lbs.
$a_{1n}, a_{2n}, \cdots, a_{10n}$	Influence coefficients for shaft section n, eq. (H.46)
B	Combined damping of rotor bearings, lb-sec/inch
B_o	Average flux density, Lines/inch^2 or Kilolines/inch^2
B_N, B_S	Flux density at north poles or south poles, Lines/inch^2
$B_{xx}, B_{xy}, B_{yx}, B_{yy}$	Damping coefficients of bearing, lbs-sec/inch
C	Radial airgap or clearance, inch
E	Youngs modulus, lbs/inch^2
E_A	Effective or voltmeter value of line voltage, volts
E_f	Voltage of field coil, volts
E_k	Rotor impedance matrix at k'th harmonic
e	Eccentricity between rotor center and stator center, inch
F	Vector $= \left\{\begin{matrix} F_x \\ F_y \\ T_x \\ T_y \end{matrix}\right\}$
F_{Nx}, F_{Ny}	x and y-component of magnetic force due to north poles, lbs.
F_{Sx}, F_{Sy}	x and y-component of magnetic force due to south poles, lbs.
F_x, F_y	x and y-component of total magnetic force in centerplane of alternator, lbs.
$\mathcal{F}$	Magnetomotive force
$\mathcal{F}_A$	Magnetomotive force of armature reaction
$\mathcal{F}_{Ad}, \mathcal{F}_{Aq}$	Demagnetizing and crossmagnetizing component of armature reaction
$\mathcal{F}_f$	Magnetomotive force of field coil
$\mathcal{F}_N, \mathcal{F}_S$	Drop in magnetomotive force across the north or the south poles

f	Drop in magnetomotive force across airgap in heteropolar generator
f_o	$= 1/2 \frac{N_f}{R_f} E_f$
f_d	$= 2 \frac{|3_{\Lambda d}|}{3_f}$, eq. (E.2), Appendix V
f_q	$= 2 \frac{|3_{\Lambda q}|}{3_f}$, eq. (E.3), Appendix V
$f_{c1}, f_{s1}, f_{c2},$ —	Harmonics of (f/f_o)
G	Shear modulus, lbs/inch^2
G	$= 1/2 \, (Q + iq)$
H	$= 1/2 \, (Q - iq)$
h	Airgap at pole for eccentric rotor, inch
h_{kj}	Airgap at the j'th stator tooth of the k'th pole, inch
I	Transverse area moment of inertia of shaft section, inch^4
I_A	Effective or ammeter value of armature current, amp
I_{pn}	Polar mass moment of inertia of rotor station mass, lbs-inch-sec^2
I_{Tn}	Transverse mass moment of inertia of rotor station mass, lbs-inch-sec^2
I_o, I_1, I_2	Defined by eqs. (E-20) to (E.22), Appendix V
i	$= \sqrt{-1}$, the imaginary unit
i_A	Armature current, amp.
i_f	Field coil current, amp
K	Combined stiffness of rotor bearings, lbs/inch
K_d	Distribution factor for armature winding
K_p	Pitch factor for armature winding
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Spring coefficients of bearing, lbs/inch
L_A	Inductance of line circuit, Henries
L_f	Inductance of field coil circuit, Henries
L_p	Distance between pole planes in homopolar generator, inch

x

ℓ	Length of generator stator, inch
ℓ	Rotor span between bearings, inch
ℓ_n	Length of shaft section n, inch
M_x, M_y	x and y-component of rotor bending moment to the left of a rotor mass station, lbs-inch
M'_x, M'_y	x and y-component of rotor bending moment to the right of a rotor mass station, lbs-inch
m	Mass of a rigid, symmetric rotor, lbs-sec^2/inch
N_A	Number of turns of a armature winding
N_f	Number of turns of a field coil
n	Number of generator north poles (=number of south poles)
n_r	Number of rotor teeth in the heteropolar generator
n_s	One half the number of stator teeth per pole in the heteropolar generator
P	$= \dfrac{\mu\, n_s A r}{2C}$, permeance of airgaps in heteropolar generator
pf	Power factor $= \cos\psi$
$\overline{Q}$	$= \frac{1}{72,130,000}$, conversion factor to get magnetic force in lbs.
Q, q	Matrices of magnetic force gradients, see eqs. (J.9) and (J.10), Appendix IX
Q_o	Negative stiffness of the static magnetic forces, lbs/inch
Q'_o	Negative moment stiffness of the static magnetic moments, lbs-inch/radian
Q_1	Stiffness of timevarying magnetic force, lbs/inch
Q_{ref}	Reference value used to represent the magnetic force stiffness in the stability calculation, see eq. (L.1)
$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}$	Cosine components of the radial stiffness of the magnetic forces, lbs/inch

$Q_{x\theta}, Q_{x\varphi}, Q_{y\theta}, Q_{y\varphi}$	Cosine components of the radial stiffness of the magnetic forces, lbs/radians
$Q_{\theta x}, Q_{\theta y}, Q_{\varphi x}, Q_{\varphi y}$	Cosine components of the angular stiffness of the magnetic forces, lbs-inch/inch
$Q_{\theta\theta}, Q_{\theta\varphi}, Q_{\varphi\theta}, Q_{\varphi\varphi}$	Cosine components of the angular stiffness of the magnetic forces, lbs-inch/radian
$q_{xx}, q_{xy}, q_{yx}, q_{yy}$	Sine components of the radial stiffness of the magnetic forces, lbs/inch
$q_{x\theta}, q_{x\varphi}, q_{y\theta}, q_{y\varphi}$	Sine components of the radial stiffness of the magnetic forces, lbs/radian
$q_{\theta x}, q_{\theta y}, q_{\varphi x}, q_{\varphi y}$	Sine components of the angular stiffness of the magnetic forces, lbs-inch/inch
$q_{\theta\theta}, q_{\theta\varphi}, q_{\varphi\theta}, q_{\varphi\varphi}$	Sine components of the angular stiffness of the magnetic forces, lbs-inch/radian
R_A	Resistance of the line circuit, ohms
R_f	Resistance of the field coil circuit, ohms
$\mathcal{R}$	Reluctance
$\mathcal{R}_N, \mathcal{R}_S$	Combined reluctance of the airgaps at the north and south poles
$\mathcal{R}_{Nk}, \mathcal{R}_{Sk}$	Reluctance of the airgap at the k'th north or south pole
$\mathcal{R}_{ak}, \mathcal{R}_{bk}$	Reluctances of the two airgaps at the k'th pole in the heteropolar generator
$\mathcal{R}_I$	Reluctance of the flux path through the stator, see eq. (8)
$\mathcal{R}_R$	Reluctance of the flux path through the rotor, see eq. (9)
$\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_4$	Reluctances of airgaps 1,2,3 and 4 in the Two-Coil Lundell generator, see fig. 3
r	Radius , inch
S_k	Matrix used in the solution of the stability or response calculation, see eqs. (J.20) and (K.17)

S_o	Value of S_k for $k=0$, see eqs. (J.23) and (K.21)
S_{co}, S_{so}	$S_{co} + i S_{so} = S_o$
T_x, T_y	x and y-component of magnetic moment in centerplane of alternator, lbs-inch
t	Time, seconds
V_x, V_y	x and y-component of rotor shear force to the left of a rotor mass station, lbs
V'_x, V'_y	x and y-component of rotor shear force to the right of a rotor mass station, lbs
X	Vector representing the rotor amplitudes and slopes at the alternator centerplane, see eq. (H.69)
X_k	The k'th harmonic of X, see eqs. (J.7) and (K.6)
X_{ck}, X_{sk}	$X_k = X_{ck} + i X_{sk}$, see eqs. (J.8) and (K.7)
X_1	Vector representing the amplitudes and slopes at the first rotor station, see eq. (H.63)
x, y	Rotor amplitudes , inch
x, y	Rotor amplitudes in the centerplane of the alternator, inch
x_o, y_o	x and y-component of the static rotor eccentricity in the centerplane of the alternator, inch
x'_o	Initial rotor eccentricity, inch
x_{ck}, x_{sk}	Cosine and sine component of the k'th harmonic of the rotor x-amplitude, inch
x_{ck}, y_{sk}	Cosine and sine component of the k'th harmonic of the rotor y amplitude, inch
x_N, y_N	Rotor amplitudes in the plane of the north poles, inch
x_S, y_S	Rotor amplitudes in the plane of the south poles, inch
x_n, y_n	Rotor amplitudes at rotor mass station, n, inch

Y_A	Admittance of the line circuit, ohms^{-1}
Y_f	Admittance of the field coil circuit, ohms^{-1}
Z_A	Impedance of the line circuit, ohms
Z_f	Impedance of the field coil circuit, ohms
z	Coordinate along the rotor axis, inch
α	Angle between x-axis and direction of displacement
α	Cross-sectional factor for shear stress
β	$= \left[\nu^2 \rho A / EI\right]^{1/4}$, inch^{-1}
β_1, β_2	Defined by eq. (H.32), Appendix VIII, inch^{-1}
γ_j	$= \pm (4j-1)\dfrac{\pi}{2n_r}$
δ	$= \left[EI / 2\alpha AG\right]^{1/2}$, inch
δ	Power angle for three-phase winding
ε	$= e/C$, eccentricity ratio
Θ	$= \dfrac{dx}{dz}$, slope of rotor in x-plane, inch/inch
Θ	Slope of rotor in x-plane at centerplane of alternator, inch/inch
Θ_0	Rotor misalignment angle in x-plane, inch/inch
Θ_n	Slope of rotor in x-plane at rotor station n, inch/inch
$\varkappa_{11}, \lambda_{11}, \varkappa_{12}, \lambda_{12}, --$	Elements of rotor impedance matrix E_k
$\varkappa_k$	$= K - Q_0 - (k\nu)^2 m$
λ_k	$= k\nu B$
λ_1 , λ_2	$= \ell_n \beta_1 , = \ell_n \beta_2$
μ	Permeability of air
ν	Frequency, radians/sec
ρ	Mass density of shaft material, lbs-sec^2/in^4

φ	$= \dfrac{dy}{dz}$, slope of rotor in y-plane, inch/inch
φ	Slope of rotor in y-plane at centerplane of alternator, inch/inch
φ_0	Rotor misalignment angle in y-plane, inch/inch
φ_n	Slope of rotor in y-plane at rotor station n, inch/inch
φ	Magnetic flux, lines
$\varphi_{Nk}, \varphi_{Sk}$	Flux at the k'th north pole or south pole, lines
$\varphi_{ak}, \varphi_{bk}$	Flux components at the k'th pole in heteropolar generator, lines.
φ_1, φ_3	Flux components for the two-coil Lundell generator, lines
Ω	Frequency of timevarying magnetic forces, radians/sec
ω	Angular speed of rotor, radians/sec
ω_c	Critical speed of symmetric, rigid rotor, radians/sec
$\omega_x, \omega_y, \omega_\theta, \omega_\varphi$	Critical speeds of a rigid rotor, radians/sec

Subscripts

A	Armature and line circuit
a,b,c	Phase a,b,c of three phase winding, Appendix IV
f	Field coil circuit
c,s	Cosine and sine component (real and imaginary part)
j	Stator tooth number in heteropolar generator
k	Generator pole number
k	Frequency harmonic number
n	Rotor station number
N,S	North pole, south pole
x,y	In the x-direction or the y-direction
Θ, φ	In the Θ-direction or the φ-direction

Indices

j	Stator tooth number in heteropolar generator
k	Generator pole number
k	Frequency harmonic number
n	Rotor station number

INTRODUCTION

In the development of high speed electrical machinery for space power plants
and compact power conversion machinery it has been found that the rotor may
exhibit unsafe large amplitude vibration under certain operating conditions.
This vibration is caused by the interaction between the rotor and the magnetic
forces in the airgaps of the electrical machinery. It is further accentuated
by the fact that the rotor bearings in this type of machinery employ low viscosity
fluids or gas as a lubricant whereby the bearing stiffness and damping is smaller
than for conventional bearings.

The application of alternators to this type of machinery is still in an early
development phase and the experience with the vibration problem derives so far
from machinery employing electrical motors. However, because of the close
similarities between alternators and electrical motors it is to be expected that
alternators may develop the same kind of vibration problems as previously found in
motor applications. For this reason an analytical investigation of the problem
as it could occur in alternators has been undertaken. It is the purpose of this
volume to present an analysis of the magnetic forces in three brushless alternator
types and to describe two computer programs which have been written to calculate
the stability and vibratory response of an alternator rotor subjected to magnetic
forces.

The three brushless generators investigated are: 1) the homopolar generator,
2) the heteropolar inductor generator, and 3)   the two-coil Lundell generator.
They are shown schematically in Figs. 1 to 3. Formulas are given from which the
magnetic forces and moments can be calculated directly once the dimensions and
operating conditions of the generator are known. Numerical examples are given
to illustrate the practical use of the formulas.

Two computer programs have been written for calculating the dynamical performance
of the alternator rotor with the imposed magnetic forces. The manuals for the
programs, containing listings of the programs and the detailed instructions of
how to use the programs and prepare the input data, are given in Appendices

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XI and XII. The first program examines the stability of the alternator rotor to the generator magnetic forces, and the second program calculates the resulting amplitude response of the rotor when the axis of the rotor does not coincide with the magnetic axis of the generator stator. The programs are quite general and apply to any rotor or bearing configuration.

Because of the large number of parameters involved it is not possible to give any general results or design charts. However, from the few sample calculations performed and from certain simplified analyses, indications are that for most applications the rotor vibrations are small and the stability margin is good. On the other hand, if the operating conditions are such that the magnetic force frequency can excite a resonance of the rotor-bearing system, large rotor amplitudes may result and, furthermore, the stability margin may become unacceptable. A detailed calculation is required under these circumstances, and the methods presented in this volume provide the means for performing such calculations.

GENERAL DISCUSSION

The problem of electromagnetically induced rotor vibrations was first encountered
in a gas bearing supported, electrical motor driven compressor. Little was known
about the causes of the vibrations and only by a trial test procedure were suffi-
cient modifications made to the design that the unit performed satisfactorily.
In two later applications, also motor driven compressors, serious vibrations were
again encountered and, as in the first case, the problem could only be overcome
by making trial modifications to the design until the vibrations were eliminated.
In one of the cases, the problem was solved by changing the method of field exci-
tation, proving that the problem definitely is caused by the electromagnetic forces.

When the need arose for incorporating alternators into this type of equipment it
was natural that there were apprehensions about encountering the same vibration
problems as already experienced with electrical motors. It has, therefore, been
decided to undertake an investigation of the problem as it applies to alternators
so that some design information is made available at an early state in the develop-
ment of alternators for space power plants. Since the available experience with
the problem is all based on electrical motors, the investigation is analytical
and there are no test data to compare with.

The investigation falls naturally into two parts: a) a study of the magnetic
forces in the alternator to establish methods and formulas from which the forces
can be calculated, and b) the development of computational methods to determine
the effect of the magnetic forces on the dynamics of the rotor.

The forces acting on the alternator rotor derive from the magnetic pull of the
stator which is proportional to the square of the flux density in the airgap between
the rotor and the stator. Hence, if the flux distribution is uniform around the
rotor circumference the resultant magnetic force is zero. However, for the alter-
nator to generate power it is necessary that the flux density varies around the
circumference and, also, it must turn with the rotor. Even so, as long as the
distribution is symmetric, no net forces will act on the rotor, but if the rotor
is eccentric with respect to the stator, dissymmetrics are introduced in the

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flux distribution causing a resultant force on the rotor. To illustrate, consider a simple case of a four pole homopolar generator which is studied in more detail in Appendices I and V. The rotor has two north poles and two south poles which are not in the same plane (see also figure 1):
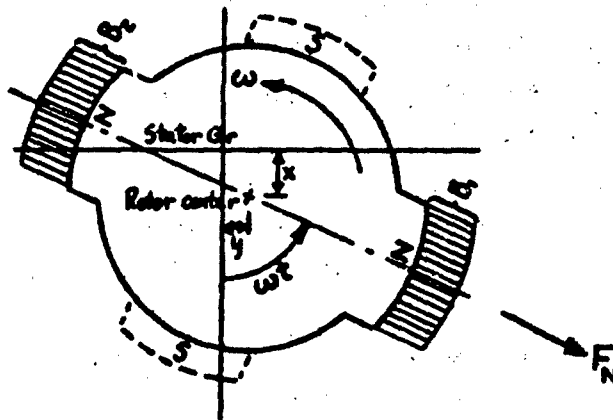
The flux density at the first north pole is B_1, kilolines per sq. inch, and at the second north pole it is B_2. If the area of a pole is A inch^2, the net force acting on the rotor due to the north poles is:

$$F_N = \frac{A}{72} \left(B_1^2 - B_2^2 \right) \tag{1}$$

where the factor $\frac{1}{72}$ is introduced to get F_N in lbs. when B_1 and B_2 are in kilolines per sq. inch and A is in sq. inch.

When the rotor is concentric within the stator, the radial airgap at the poles is C, inch, and the average flux density is B_0. In that case $B_1 = B_2 = B_0$ and the net force is zero (i.e. $F_N = 0$, see eq. (1)). However, when the rotor is eccentric such that its center has the coordinates x and y with respect to the center of the stator (see the figure above), the airgaps at the two poles are not the same. The flux density is inversely proportional to the airgap, and assuming the rotor eccentricity to be small compared to the radial gap C, the airgaps at the two poles can be expressed as:

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airgap at the first north pole: $h_1 = C\left[1 - \frac{x}{C}\cos(\omega t) - \frac{y}{C}\sin(\omega t)\right]$

$$(2)$$

airgap at the second north pole: $h_2 = C\left[1 + \frac{x}{C}\cos(\omega t) + \frac{y}{C}\sin(\omega t)\right]$

ω is the angular speed of the rotor, radians/sec, such that (ωt) gives the angle at time t between the x-axis and the line through the poles. Since it is assumed that the ratios $\frac{x}{C}$ and $\frac{y}{C}$ are small compared to 1, the flux densities become:

$$B_1 = \frac{C}{h_1} B_0 = B_0\left[1 + \frac{x}{C}\cos(\omega t) + \frac{y}{C}\sin(\omega t)\right]$$

$$(3)$$

$$B_2 = \frac{C}{h_2} B_0 = B_0\left[1 - \frac{x}{C}\cos(\omega t) - \frac{y}{C}\sin(\omega t)\right]$$

from which:

$$B_1^2 = B_0^2\left[1 + 2\frac{x}{C}\cos(\omega t) + 2\frac{y}{C}\sin(\omega t)\right]$$

$$B_2^2 = B_0^2\left[1 - 2\frac{x}{C}\cos(\omega t) - 2\frac{y}{C}\sin(\omega t)\right]$$

The net force acting on the rotor is then determined from eq. (1) as:

$$F_N = 4\frac{A B_0^2}{\gamma_2 C}\left[x\cos(\omega t) + y\sin(\omega t)\right]$$

$$(4)$$

This force follows the rotor as it turns. Its components in the fixed x-y-coordinate system are:

$$F_{Nx} = F_N\cos(\omega t) = 2\frac{A B_0^2}{\gamma_2 C}\left[x(1+\cos(2\omega t)) + y\sin(2\omega t)\right]$$

$$(5)$$

$$F_{Ny} = F_N\sin(\omega t) = 2\frac{A B_0^2}{\gamma_2 C}\left[x\sin(2\omega t) + y(1-\cos(2\omega t))\right]$$

It is seen that the force is directly proportional to the eccentricities x and y (this is, of course, only true when x and y are reasonably small compared to the radial airgap C). Hence, the magnetic force acts as a negative spring force (negative because the force acts in the same direction as the displacement; it does not oppose the displacement as a mechanical spring would do). It can further be observed that the force contains two parts, one part which for given eccentricities x and y is constant, and one part which varies periodically with a frequency of twice the rotor speed. The constant part of the force acts on the rotor simply as a negative, static spring, but the timevarying part can force the rotor to whirl and even induce instability as discussed

later. All generator types produce magnetic forces of the same general form as
shown above although the timevarying part may be absent in some cases. Of the three
generator types studies, the four pole homopolar generator and the heteropolar
inductor generator under load have magnetic forces with timevarying components,
whereas the homopolar generator with more than four poles and the two-coil Lundell
generator only have static force components.

To complete the above example of the magnetic forces in the four pole homopolar
generator, where only the forces acting on the north poles have been considered
so far, the forces acting on the south poles, F_{Sx} and F_{Sy} , can be obtained
simply by observing that the south poles lag the north poles by 90 degrees.
Hence, by replacing (ωt) by ($\omega t - 90$) in eqs. (5), the forces on the south
poles become:

$$F_{Sx} = 2 \frac{A B_a^2}{72 C} \left[x(1-\cos(2\omega t)) - y \sin(2\omega t) \right]$$

$$F_{Sy} = 2 \frac{A B_a^2}{72 C} \left[-x \sin(2\omega t) + y(1+\cos(2\omega t)) \right]$$

(6)

Thus, if the north poles and the south poles are in the same plane, the net forces
acting on the rotor, namely $F_x = (F_{Nx} + F_{Sx})$ and $F_c = (F_{Ny} + F_{Sy})$, are in-
dependent of time. Only, when the pole planes do not coincide, is there a possi-
bility of a timevarying force or a timevarying moment. For details, see Appendix I.

This simple analysis illustrates the general character of the more detailed analysis
employed in Appendices I to VI to derive the formulas for calculating the magnetic
forces. The analyses are concerned with the fundamental harmonic of the forces
and do not consider the contributions from such factors as higher harmonics in the
flux wave or higher harmonics in the flux density distribution caused by edge
effects, slotting or pole shape. It will normally be found that such factors have
negligible influence on the net forces although they can cause appreciable local
forces. They are basically unaffected by rotor eccentricity and, therefore, cancel
out when the net force is obtained. The effect of generator load, on the other
hand, cannot be ignored and is included in the analysis. When the alternator
operates under load, the armature windings produce a magnetomotive force, commonly
known as the armature reaction, which opposes the flux direction of the main
field. Thus, the general effect of the armature reaction is to reduce the magnetic
forces and, at the same time, it may also introduce new timevarying force components

which have the frequency of the line current or harmonics thereof.

The analysis ignores saturation effects in the iron and assumes that all of
the reluctance in the magnetic flux path occurs in the airgaps at the poles.
In practice this assumption is not valid where the iron operates close to satu-
ration. The effect of saturation is to reduce the magnetic force values
obtained on the basis of unsaturated iron. To illustrate, return to the homo-
polar generator analyzed above. As shown in Figure 1, the field coil is between
the two pole planes. The flux starts in the rotor at the north poles, passes
the airgaps of the north poles, goes through the stator iron and returns to
the rotor via the airgaps of the south poles. The combined reluctance of the
two airgaps of the north poles is (two reluctances in parallel):

$$\mathcal{R}_N \cong \frac{C}{2\mu A} \tag{7}$$

where C is the radial airgap, A is the pole area and μ is the permeability
of air. The reluctance $\mathcal{R}_S$ of the airgaps at the south poles is the same,
i.e. $\mathcal{R}_S = \mathcal{R}_N$. The reluctance $\mathcal{R}_I$ of the flux path through the stator iron
depends on how the flux is distributed over the cross-sections of the path.
Symbolically the reluctance may be written:

$$\mathcal{R}_I = \int_0^{l_I} \frac{dz}{\mu_I A_I} \cong \frac{l_I}{(\mu_I A_I)_{average}} \tag{8}$$

where l_I represents the total length of the flux path, z is the coordinate
along the path, A_I is the cross-sectional flux area which depends on z
and μ_I is the permeability of the stator iron. μ_I is a function of the local
flux density and, thus, is a function of z (it actually also varies over the
cross-sectional area). The calculation of $\mathcal{R}_I$ is rather complicated since the
permeability is a non-linear function of the flux density, thereby making it
necessary to compute the detailed flux distribution in order to find the effective
overall reluctance $\mathcal{R}_I$ of the flux path.

The reluctance $\mathcal{R}_R$ of the flux path through the rotor can be represented in
a similar way:

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$$R_R \cong \frac{l_R}{(\mu_R A_R)_{average}} \tag{9}$$

where the meaning of the symbols is the same as above. The total reluctance of the flux path is the sum of the four reluctances: $R_N + R_S + R_I + R_S$. Thus, if the field coil produces a magnetomotive force $\mathcal{F}_f$, the drop in mmf, $\mathcal{F}_N$, across the airgaps at the north poles become:

$$\mathcal{F}_N = \frac{R_N}{R_N + R_S + R_I + R_R} \mathcal{F}_f \tag{10}$$

where $R_S = R_N$. If saturation effects are ignored, then $R_I = R_R = 0$ and $\mathcal{F}_N = \frac{1}{2} \mathcal{F}_f$. The actual drop in mmf is smaller, causing a proportional reduction in flux density and, therefore, a parallel reduction in the magnetic forces. In this way the effect of saturation can be included by multiplying the force values obtained on the basis of unsaturated iron by the factor: $[R_N / (2R_N + R_I + R_R)]^2$. It should be emphasized, however, that this adjustment is only necessary if the flux density has been calculated on the basis of unsaturated iron. If instead the actual flux density, B_0 , in the airgaps at the poles is known and used directly in the formulas given in the following sections, the effect of saturation is already included (because the effect is included in B_0).

Having determined the magnetic forces, their effect on the rotor can be studied. It should first be noted that that part of the forces which does not vary with time, has only a "passive" effect. It can be represented simply by negative springs in parallel with the stiffnesses of the rotor bearings and the stiffness of the rotor shaft. It is obvious that if this negative stiffness is large enough to offset the combined rotor-bearing stiffness, then the rotor will be statically unstable or, in other words, the magnetic forces are so large that the rotor is simply pulled up against the stator. This case is of academic interest only or, at least, it is readily checked and does not require any special investigation. Hence, assuming that the system is statically stable, the time-independent components of the magnetic forces can be considered an integral part of the rotor-bearing system in which they are included simply as another rotor bearing, although with a negative stiffness. In the analysis this negative stiffness is called Q_0 ,

lbs/inch, and in addition allowance is made for a similar negative moment stiffness Q_o' , lbs-inch/radian. They must be specified in the input to the rotor computer programs.

Turning next to the timevarying components of the magnetic forces, they can influence the rotor in two ways: they may induce instability and, furthermore, if there is any built-in eccentricity between the rotor and the stator, they can force the rotor to whirl. Using again the previous example of the 4 pole homopolar generator, let the rotor displacements measured from the center of the alternator stator be x_N and y_N in the plane of the north poles, and x_S and y_S in the plane of the south poles. Then the timevarying components of the magnetic forces can be written (from eqs. (5) and (6)):

$$(F_{Nx})_{timevarying} = \frac{A B_o^2}{36C} \left[x_N \cos(2\omega t) + y_N \sin(2\omega t) \right]$$

$$(F_{Ny})_{timevarying} = \frac{A B_o^2}{36C} \left[x_N \sin(2\omega t) - y_N \cos(2\omega t) \right]$$

(11)

$$(F_{Sx})_{timevarying} = - \frac{A B_o^2}{36C} \left[x_S \cos(2\omega t) + y_S \sin(2\omega t) \right]$$

$$(F_{Sy})_{timevarying} = - \frac{A B_o^2}{36C} \left[x_S \sin(2\omega t) - y_S \cos(2\omega t) \right]$$

Let the distance between the two pole planes be Lp, inch. Then the slopes of the rotor are:

$$\Theta = \frac{1}{L_p} \left(x_N - x_S \right)$$

(12)

$$\varphi = \frac{1}{L_p} \left(y_N - y_S \right)$$

The rotor displacements at the center plane of the alternator, midway between the two pole planes, are:

$$x = \tfrac{1}{2} \left(x_N + x_S \right)$$

(13)

$$y = \tfrac{1}{2} \left(y_N + y_S \right)$$

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The forces F_x and F_y, and the moments T_x and T_y acting on the rotor at the centerplane of the alternator are:

$$F_x = F_{Nx} + F_{Sx}$$
$$F_y = F_{Ny} + F_{Sy}$$
$$T_x = \tfrac{1}{2} L_p (F_{Nx} - F_{Sx}) \tag{14}$$
$$T_y = \tfrac{1}{2} L_p (F_{Ny} - F_{Sy})$$

Substituting eqs. (11) into eqs (14) and making use of eqs. (12) and (13) yields:

$$F_x = \frac{A B_o^2}{36 C} L_p \left[\theta \cdot \cos(2\omega t) + \varphi \sin(2\omega t) \right]$$
$$F_y = \frac{A B_o^2}{36 C} L_p \left[\theta \cdot \sin(2\omega t) - \varphi \cos(2\omega t) \right]$$
$$T_x = \frac{A B_o^2}{36 C} L_p \left[x \cdot \cos(2\omega t) + y \sin(2\omega t) \right] \tag{15}$$
$$T_y = \frac{A B_o^2}{36 C} L_p \left[x \sin(2\omega t) - y \cos(2\omega t) \right]$$

Now, assume that the rotor starts to whirl in a closed orbit such that x, y, θ, and φ represent harmonic oscillations. Then the magnetic forces perform work on the rotor and, if this work is integrated over one cycle to determine the net energy, it will be found, that if the rotor motion is periodic with a fundamental frequency of either ω or 2ω, the possibility exists of the energy input to the rotor being positive. In other words, energy can be transferred from the magnetic field to the rotor motion. It obviously depends on the phase relationships between the magnetic forces and the rotor motion if the energy transfer to the rotor will be positive or negative, but if the energy is positive, the rotor motion will actually grow and the rotor is unstable. If the energy is zero, the rotor motion will persist indefinitely and the system is on the threshold of instability. The phase relationship between the magnetic forces and the rotor motion is governed by the stiffness, damping and inertia properties of the rotor-bearing system and for an actual rotor it is necessary to perform the detailed calculations on a computer. The computer program for such a calculation is described in details in Appendix XI. Although the program does not actually check the rotor stability by means of the outlined energy method, the employed method is equivalent and the basic analysis is described in Appendix IX. The program calculates the threshold of instability as the zero-point of either of two determinants in complete analogy to the above development where the rotor is on the threshold of instability when

the energy input is zero which may happen when the fundamental frequency is
either ω or 2ω (in fact, the two determinants corresponds to rotor motions
with these two fundamental frequencies). In searching for the zero points of
the determinants (i.e. the threshold of instability), the rotor speed is kept
fixed and the magnetic forces are varied over a specified range. Once the
threshold has been found, it is immediately checked if the actual magnetic forces
puts the rotor in a stable or an unstable zone of operation.

This form of instability is normally classified as a Mathieu type of instability
(references 1 and 2). However, the classical Mathieu equation represents, in
this context, a rigid, symmetrical rotor with only one critical speed, with only
one amplitude direction and with no damping in the system. A stability map for
such a system, although with damping included, is shown in figure 5. It will be
discussed in detail in the chapter entitled "The Concept of Stability and Response
of a Rotor with Magnetic Forces." Most rotors, however, can not be represented
in this simplified fashion. A typical rotor is not entirely symmetric and of
greater importance, it has many critical speeds. Furthermore, the Mathieu equation
allows only for one form of the magnetic forces which, as an example, cannot be
made to represent the forces in a four pole homopolar generator. The developed
computer program is far more general and can treat any arbitrary rotor with all
its resonances, and also makes it possible to specify magnetic forces of any
form desired.

In writing the program it has been considered to admit more than one frequency
in the magnetic forces (in the language of the literature: to go from a Mathieu
equation to a Hill equation). However, the study of the three selected generator
types does not indicate that higher harmonics of the fundamental frequency are
important. Hence, only the force components of the fundamental harmonic are
treated by the computer program. Furthermore, to include higher harmonics would
drastically increase the computer time. From the point-of-view of the analysis
or writing the computer program, it is just as easy to treat any number of
frequencies than just one but it is felt, at this stage, that it would be un-
justified because of the computer time.

It should be emphasized that it is not a simple routine matter to perform a stability calculation. It requires some pre-knowledge of the characteristics of the rotor-bearings system (notably where the critical speeds are) in order to interpret the results of the calculations correctly, and it is necessary to perform calculations not only at the operating speed but over a sufficient range of speeds that a stability map can be established. These problems are discussed at length in the chapter entitled: "Discussion on Performing Stability Calculations."

The rotor response calculation is more readily performed than the stability calculation. Let eqs. (15) be representative of the timevarying magnetic forces and assume that there is a built-in eccentricity between the rotor centerline and the stator axis such that, without the rotor whirling, the center of the rotor has the coordinates X_0 and y_0 with respect to the center of the stator, measured in the centerplane of the alternator. Furthermore, the axis of the rotor is mis-aligned with respect to the stator axis by the angles Θ_0 and φ_0. Then, as seen from eqs. (15), forces and moments will act on the rotor:

$$\left(F_x\right)_0 = \frac{AB_0^2}{36C} L_p \left[\Theta_0 \cos(2\omega t) + \varphi_0 \sin(2\omega t)\right]$$

$$\left(F_y\right)_0 = \frac{AB_0^2}{36C} L_p \left[\Theta_0 \sin(2\omega t) - \varphi_0 \cos(2\omega t)\right]$$

$$\left(T_x\right)_0 = \frac{AB_0^2}{36C} L_p \left[x_0 \cos(2\omega t) + y_0 \sin(2\omega t)\right]$$

$$\left(T_y\right)_0 = \frac{AB_0^2}{36C} L_p \left[x_0 \sin(2\omega t) - y_0 \sin(2\omega t)\right]$$

(16)

These forces and moments will obviously force the rotor to whirl. Since the forces have a frequency of 2ω, radians/sec, it is to be expected that the rotor will whirl with the same frequency, i.e. the rotor vibrations will be at twice per revolution instead of the synchronous vibration encountered when the rotor has a mechanical unbalance. In general it will be found that this will be the predominant frequency of the vibration. However, as shown by eq. (15), the magnetic forces depend on the rotor amplitude. Eqs. (16) only represents that part of the forces which is induced by the built-in eccentricity. The remaining part of the magnetic forces, namely the difference between eqs. (15) and (16), will interact with the induced forces via the rotor-bearing system and will cause the rotor to respond not only with a frequency of 2ω, but also with the frequencies $4\omega, 6\omega, 8\omega$ and so on. This calculation is performed by means of the computer program described in details in Appendix XII.The basic analysis is contained in Appendix X. As in

the stability computer program, the rotor may be any arbitrary flexible rotor
with several bearings, and the form of the magnetic forces is quite general.
To calculate the response, the built-in eccentricities between the rotor and the
stator must, of course, be specified. A more detailed discussion of this type
of calculation is given in the chapter: "Discussion on Performing Response
Calculations."

In summary it can be said that the analyses, the formulas for calculating the
magnetic forces in the generator and the two rotor dynamic computer programs
are quite general, and together they provide adequate means for a comprehensive
check of the performance of a proposed alternator rotor-bearing system design.
It should be noted, however, that whereas the presented analyses and the compu-
tational methods are believed to be sound, there are no test data or actual
measurements available against which the theoretical predictions can be checked
and compared. Furthermore, in those applications where the magnetic force
gradients are small compared to the combined rotor-bearing stiffness (say, less
than 30 percent), the two rotor computer programs are unnecessarily "sophisticated"
and unjustifiably complex. They give correct results but in far more detail
than required for design purposes. On the other hand it is felt, that as long as
no real practical experience is available to serve as a guide, it is safer to use
calculation methods which are generally applicable although for any particular
application much of the generated information may prove to be of limited practical
significance. At this early time in the development of designing alternators
for space power plants it is not possible to predict with any accuracy what
future requirements may demand and viewed in that context the presented methods
should be able to serve their purpose.

THE MAGNETIC FORCES OF THREE BRUSHLESS GENERATOR TYPES

Three brushless generator types are investigated: the homopolar generator, the heteropolar inductor generator and the two-coil Lundell generator. They are shown schematically in Figures 1 to 3. The magnetic forces produced by these generators have been derived for the generators operating without and with load and the analyses are given in detail in Appendices I to III. Since manufactured generators differ widely in construction (notably in the way they are wound), even if they are of the same type, the analyses are kept general, not specific. The objective of the analyses is to derive simple formulas from which the magnetic forces can be calculated with sufficient accuracy for engineering purposes. Only the fundamental harmonic of the forces are considered and such factors as higher harmonics in the flux wave or stator and rotor slotting are disregarded. The most serious assumption is that saturation effects are ignored. However, saturation will reduce the magnitude of the forces and the developed formulas will,' therefore, give too large forces. Thus, the rotor calculations will be conservative and actually have a built-in safety factor. It should be noted, on the other hand, that in an actual generator the eccentricity between the stator and the rotor is not known with too high a degree of accuracy, so that a safety factor is required anyway.

The formulas for calculating the magnetic forces are set up to conform with the required input format to the rotor stability and the rotor response computer programs. Hence, the calculated numerical values can be used directly as input to either of the two computer programs. To explain the input format, let x and y be the amplitudes of the rotor in the centerplane of the generator, and let Θ and φ be the corresponding slopes of the rotor (i.e. $\Theta = \frac{dx}{dz}$ and $\varphi = \frac{dy}{dz}$, taken at the centerplane of the generator, where z is the coordinate along the rotor axis). The magnetic forces have the two force components: F_x and F_y, and the two moment components: T_x and T_y. The forces are assumed to be proportional to the amplitudes and slopes of the rotor:

$$
\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \begin{Bmatrix} Q_o x \\ Q_o y \\ Q_o' \Theta \\ Q_o' \varphi \end{Bmatrix} - \left[\begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{x\Theta} & Q_{x\varphi} \\ Q_{yx} & Q_{yy} & Q_{y\Theta} & Q_{y\varphi} \\ Q_{\Theta x} & Q_{\Theta y} & Q_{\Theta\Theta} & Q_{\Theta\varphi} \\ Q_{\varphi x} & Q_{\varphi y} & Q_{\varphi\Theta} & Q_{\varphi\varphi} \end{Bmatrix} \cos(\Omega t) - \begin{Bmatrix} q_{xx} & q_{xy} & q_{x\Theta} & q_{x\varphi} \\ q_{yx} & q_{yy} & q_{y\Theta} & q_{y\varphi} \\ q_{\Theta x} & q_{\Theta y} & q_{\Theta\Theta} & q_{\Theta\varphi} \\ q_{\varphi x} & q_{\varphi y} & q_{\varphi\Theta} & q_{\varphi\varphi} \end{Bmatrix} \sin(\Omega t) \right] \begin{Bmatrix} x \\ y \\ \Theta \\ \varphi \end{Bmatrix} \qquad (17)
$$

Here, F_x and F_y are the magnetic forces in lbs, T_x and T_y are the magnetic moments in lbs-inch, Ω is the frequency of the magnetic forces in radians/sec., t is time in seconds, and the Q's and q's are the gradients of the magnetic forces and moments where the first index specifies the force direction and the last index gives the amplitude direction. The two computer programs require that the values of these gradients are given in the computer input. In the following it will be shown how the gradients are obtained for the generator types under study.

The 4 Pole Homopolar Generator - The magnetic forces in the homopolar generator are analyzed in Appendices I and V. There it is shown that only for the four pole generator are the magnetic forces timevarying. Thus, the rotor stability program and the rotor response program only apply to this case. For a different number of poles, there is no response or stability problem.

As shown in Figure 1, the north poles and the south poles are in separate planes. Let the distance between the two planes be Lp, inch. Then the magnetic forces and 'moments for the generator operating with no load become (see Eq. (A.46), Appendix I):

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \frac{AB_a^2}{36C}\left[\begin{Bmatrix} 2x \\ 2y \\ \frac{1}{2}L_p^2\theta \\ \frac{1}{2}L_p^2\phi \end{Bmatrix} - L_p\left[\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\cos(2\omega t) - \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\sin(2\omega t)\right]\begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix}\right] \quad (18)$$

where ω is the angular speed of the rotor in radians/sec. Hence, in terms of eq. (17):

$$Q_o = 4\frac{AB_a^2}{72C} \qquad \frac{lbs}{inch} \qquad\qquad (19)$$

$$Q_o' = \frac{AB_a^2}{72C}L_p^2 \qquad \frac{lbs \cdot inch}{radian} \qquad\qquad (20)$$

$$-Q_{x\theta} = Q_{y\phi} = -Q_{\theta x} = Q_{\phi y} = q_{x\theta} = q_{y\phi} = q_{\theta x} = q_{\phi y} = 2\frac{AB_a^2}{72C}L_p \qquad lbs \qquad (21)$$

(all other values of Q and q are zero)

The nomenclature is:

A = area of one pole, $inch^2$

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C = radial air gap at the poles, inch

L_p = distance between planes of north poles and south poles, inch.

B_o = Average flux density in the air gaps at the poles due to the field coils, kilolines/inch^2

To take an example, let the average flux density be 50 kilolines/inch^2. (This is a rather average value for generators for space power plants and similar applications). Furthermore, let the radial air gap be 0.040 inch, the length between pole planes: L_p = 4.2 inch and the pole area: A = 5.76 inch^2. Then:

$$\frac{A B_o^2}{72\,C} = \frac{5.76 \cdot (50)^2}{72 \cdot 0.040} = 5,000 \ \frac{lbs}{inch}$$

Hence:

$$Q_o = 20,000 \ \frac{lbs}{inch}$$
$$Q_o' = 88,200 \ \frac{lbs \cdot inch}{radian}$$
$$-Q_{x\theta} = Q_{y\varphi} = q_{x\theta} = q_{y\varphi} = 42,000 \ \frac{lbs}{radian} \tag{22}$$
$$-Q_{\theta x} = Q_{\varphi y} = q_{\theta x} = q_{\varphi y} = 42,000 \ \frac{lbs \cdot inch}{inch}$$

The ratio between the magnetic force frequency and the speed of the rotor is:

$$\frac{\Omega}{\omega} = 2$$

In this way all the magnetic force input data required for the two computer programs have been obtained.

When the generator is loaded, all the above values are reduced by being multiplied by the factor $(1-f_d)^2$ where:

$$f_d = \frac{12\sqrt{2}}{\pi} \ \frac{R_f}{N_f E_f} K_d K_p N_A I_A \sin(\psi + \delta) \tag{23}$$

where:

R_f = resistance of the field coil, ohms.

N_f = number of turns of the field coil

E_f = the d.c. voltage impressed in the field coils, volts

N_A = number of turns of one armature winding.

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I_A = the am-meter value of the current, amps

K_d = distribution factor ($\cong 0.96$ to 1)

K_p = pitch factor ($\cong 0.96$ to 1)

γ = power factor angle

δ = power angle.

Thus, f_d gives the ratio between the de-magnetizing component of the armature reaction and half of the mmf of the field coil. A detailed discussion is given in Appendices IV and V. To calculate f_d it is necessary to obtain the necessary data from the generator manufacturer.

The Heteropolar Inductor Generator under Load. The magnetic forces of the hetero-polar inductor generator under load are analyzed in Appendix VI. Only when the generator operates under load are the magnetic forces timedependent and, thus, only in this case can a rotor stability and a rotor response calculation be performed.

The heteropolar inductor generator produces magnetic forces only and no moments. The forces are derived in Appendix VI as:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{2nn_s A_r B_p^2}{72C} \left[(1+\tfrac{1}{2}f_{c1}^2+\tfrac{1}{2}f_{s1}^2)\begin{Bmatrix} x \\ y \end{Bmatrix} - \left[2f_{c1}\begin{Bmatrix} -1 & 0 \\ 0 & -1 \end{Bmatrix} \cos(n_p\omega t) - 2f_{s1}\begin{Bmatrix} -1 & 0 \\ 0 & -1 \end{Bmatrix} \sin(n_p\omega t)\right]\begin{Bmatrix} x \\ y \end{Bmatrix} \right] \quad (24)$$

Hence, in terms of eq. (17):

$$Q_o = \frac{2nn_s A_r B_p^2}{72C}\left[1+\tfrac{1}{2}(f_{c1}^2+f_{s1}^2)\right] \qquad \frac{lbs}{inch} \qquad (25)$$

$$Q_o' = 0 \qquad (26)$$

$$Q_{xx} = Q_{yy} = -\frac{2nn_s A_r B_p^2}{72C}2f_{c1} \qquad \frac{lbs}{inch} \qquad (27)$$

$$q_{xx} = q_{yy} = -\frac{2nn_s A_r B_p^2}{72C}2f_{s1} \qquad \frac{lbs}{inch} \qquad (28)$$

All other gradients are zero. The ratio between the magnetic force frequency Ω and the rotor speed is:

$$\frac{\Omega}{\omega} = n_r \qquad \qquad (29)$$

The nomenclature is:

$2n$ = total number of poles (n north poles and n south poles)

n_s = one half the number of stator teeth per pole

n_r = total number of rotor teeth

A_T = area of one stator tooth, inch

C = radial air gap at the poles, inch

B_o = average flux density in the air gaps at the poles due to the field coils, kilolines/inch^2

f_{c1}, f_{s1} = fundamental components of the armature reaction, dimensionless.

The method for evaluating f_{c1} and f_{s1} is given in Appendix VI. Here it is found, restricting the analysis to the first harmonic, that:

$$f_{c1} + i f_{s1} = \frac{i\nu Y_A L_c}{1 + i\nu Y_f L_f} \qquad \qquad (30)$$

where:

$i = \sqrt{-1}$

$\nu = n_r \omega$, the frequency of the magnetic forces, radians/sec

L_c = sum of the self-inductances of the 4n armature coils, Henries

L_f = the self-inductance of one field coil, Henries

$Y_f = 1/[R_f + i\nu L_f]$, the admittance of a field coil, ohms^{-1}

R_f = resistance of a field coil, ohms

$Y_A = 1/[R_A + i\nu L_A]$, the admittance of the power circuit, ohms^{-1}

R_A = resistance of the power circuit, ohms

L_A = inductance of the power circuit, Henries

Let P be the permeance of the airgaps of one half the stator teeth of a pole. Then:

$$P = \frac{\mu n_s A_T}{2C} \qquad \qquad (31)$$

where μ is the permeability of air. Then:

$$L_c = 4n \, PN_A^2 \tag{32}$$

$$L_f = \tfrac{1}{2} \, PN_f^2 \tag{33}$$

where:

   N_A = number of turns of one armature coil (there are 4 n coils)
   N_f = number of turns of one field coil

Returning to eq. (30), it can be written:

$$f_{c1} + if_{s1} = \frac{\dfrac{i\nu L_c}{R_A + i\nu L_A}}{1 + \dfrac{i\nu L_f}{R_f + i\nu L_f}} = \frac{\dfrac{L_c/L_A}{1 - i^{R_A/\nu L_A}}}{1 + \dfrac{1}{1 - i^{R_f/\nu L_f}}} \tag{34}$$

Consider the numerator. When the line current lags the linevoltage by a phase angle ψ, then the power factor, pf, is defined as:

$$pf = \cos \psi \tag{35}$$

As shown in Appendix IV eq. (D.16):

$$pf = \frac{R_A}{\sqrt{R_A^2 + (\nu L_A)^2}} \tag{36}$$

for which:

$$\frac{R_A}{\nu L_A} = \frac{pf}{\sqrt{1 - (pf)^2}} \tag{37}$$

If it assumed, furthermore, that the field coil circuit is predominantly inductive (i.e. $R_f/\nu L_f \cong 0$), then eq. (34) becomes:

$$f_{c1} + i \, f_{s1} \cong \tfrac{1}{2} \frac{L_c}{L_A} \frac{\sqrt{1 - (pf)^2}}{\sqrt{1 - (pf)^2} - i(pf)} \tag{38}$$

from which:

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$$f_{c_1} \cong \frac{1}{2} \frac{L_c}{L_A} \left[1 - (pf)^2 \right] \qquad (39)$$

$$f_{s_1} \cong \frac{1}{2} \frac{L_c}{L_A} \cdot (pf) \cdot \sqrt{1 - (pf)^2} \qquad (40)$$

These equations must be considered to be approximate only, but they probably yield sufficient accurate results for the purpose of the rotor calculations.

To illustrate the use of the expressions, consider a numerical example. Let the generator configuration be as depicted in Fig. 2. Hence, there are 4 poles with four stator teeth per pole, and the rotor has 20 teeth:

$n = 2$

$n_s = 2$

$n_r = 20$

The rotor is 6 inches long and the area of one stator tooth is 1.25 $inch^2$. The radial airgap is 0.005 inch:

$A_T = 1.25$ $inch^2$

$C = 0.005$ inch

Assume an average flux density of:

$B_o = 50$ kilolines/$inch^2$

With these numbers:

$$\frac{2 n n_s A_T B_o^2}{72 C} = \frac{2 \cdot 2 \cdot 2 \cdot 1.25 \cdot (50)^2}{72 \cdot 0.005} = 69,440 \frac{lbs}{inch}$$

Assume the power factor to be 0.8 and let the ratio between the inductance of the armature coils and the power circuit be 1:

$pf = 0.8$

$L_c / L_A = 1$

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Then, from eqs. (39) and (40):

$$f_{c1} = 0.18$$

$$f_{s1} = 0.24$$

Then, eqs. (25) to (29) yield:

$$Q_e = 75,690 \ \frac{lbs}{inch}$$

$$Q_e' = 0$$

$$Q_{xx} = Q_{yy} = 25,000 \ \frac{lbs}{inch} \tag{41}$$

$$q_{xx} = q_{yy} = 33,300 \ \frac{lbs}{inch}$$

$$\frac{\Omega}{\omega} = 20$$

These values can be used directly as input to the rotor response and the rotor stability programs.

The Two-Coil Lundell Generator. The magnetic forces for this generator are analyzed in Appendix III. It is shown that, in general, there are no time-varying magnetic forces in the two-coil Lundell generator or, if there are, they will be small since they are caused primarily by differences in pole areas. Hence, this generator is of little interest for purposes of calculating rotor stability and rotor response. The two-coil Lundell generator has the same magnetic force characteristics as the homopolar generator except that the north poles and the south poles are in the same plane. It is this factor which is responsible for elliminating the time-averaging magnetic forces.

In the preceeding chapter, a brief discussion has been given on how the magnetic forces can cause the rotor to whirl and also induce instability. A more specific discussion is, however, necessary in order to describe the basic features of the two rotor computer programs. For this purpose, consider a simplified rotor model where the rotor is rigid and symmetric. The rotor mass is m, the total bearing stiffness is K and the total bearing damping is B. If the rotor amplitude is x the magnetic forces are taken as: $(Q_0 - Q_1 \cos(\Omega t))x$ where Ω is the frequency of the magnetic forces, Q_0 and Q_1 are the gradients of the magnetic force, and t is time. The equation of motion is:

$$m \frac{d^2 x}{dt^2} + B \frac{dx}{dt} + [K - Q_0 + Q_1 \cos(\Omega t)]x = 0 \qquad (42)$$

This equation is the damped Mathieu equation. To check its stability, expand x in a Fourier series (references 1 and 2):

$$x = \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] \qquad (43)$$

where:

$$\psi = \tfrac{1}{2} \Omega t = \nu t \qquad (44)$$

$$\nu = \tfrac{1}{2} \Omega \qquad (45)$$

Substitute eq. (43) into eq. (42):

$$\sum_{k=0}^{\infty} \left\{ \left[(K - Q_0 - (k\nu)^2 m)x_{ck} - k\nu B x_{sk}\right] \cos(k\psi) - \left[(K - Q_0 - (k\nu)^2 m)x_{sk} + k\nu B x_{ck}\right] \sin(k\psi) \right\}$$
$$+ Q_1 \cos(2\psi) \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] = 0 \qquad (46)$$

With the trigonometric identities:

$$\cos(2\psi)\cos(k\psi) = \tfrac{1}{2}[\cos(k+2)\psi + \cos(k-2)\psi]$$
$$\cos(2\psi)\sin(k\psi) = \tfrac{1}{2}[\sin(k+2)\psi + \sin(k-2)\psi] \qquad (47)$$

and collecting terms in $\cos(k\psi)$ and $\sin(k\psi)$, eq. (46) gives rise to two infinite sets of simultaneous equations. Define:

$$x_k = K - Q_0 - (k\nu)^2 m = K - Q_0 - \left(\tfrac{k\Omega}{2}\right)^2 m \tag{48}$$

$$\lambda_k = k\nu\beta = \tfrac{k\Omega}{2}\beta \tag{49}$$

whereby the two sets of equations can be written:

$$\begin{cases}
\begin{bmatrix}
x_0 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ----\\
Q_1 & x_2 & -\lambda_2 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & 0 & 0 & ----\\
0 & \lambda_2 & x_2 & 0 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & 0 & ----\\
0 & \tfrac{1}{2}Q_1 & 0 & x_4 & -\lambda_4 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & ----\\
0 & 0 & \tfrac{1}{2}Q_1 & \lambda_4 & x_4 & 0 & \tfrac{1}{2}Q_1 & 0 & 0 & ----\\
0 & 0 & 0 & \tfrac{1}{2}Q_1 & 0 & x_6 & -\lambda_6 & \tfrac{1}{2}Q_1 & 0 & ----\\
0 & 0 & 0 & 0 & \tfrac{1}{2}Q_1 & \lambda_6 & x_6 & 0 & \tfrac{1}{2}Q_1 & ----\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{bmatrix}
\begin{Bmatrix}
x_{c0}\\ x_{c2}\\ x_{s2}\\ x_{c4}\\ x_{s4}\\ x_{c6}\\ x_{s6}
\end{Bmatrix} = 0
\end{cases} \tag{50}$$

$$\underline{k = 0, 2, 4, --}$$

$$\begin{cases}
\begin{bmatrix}
(x_1+\tfrac{1}{2}Q_1) & -\lambda_1 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & 0 & 0 & ----\\
\lambda_1 & (x_1-\tfrac{1}{2}Q_1) & 0 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & 0 & ----\\
\tfrac{1}{2}Q_1 & 0 & x_3 & -\lambda_3 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & ----\\
0 & \tfrac{1}{2}Q_1 & \lambda_3 & x_3 & 0 & \tfrac{1}{2}Q_1 & 0 & 0 & ----\\
0 & 0 & \tfrac{1}{2}Q_1 & 0 & x_5 & -\lambda_5 & \tfrac{1}{2}Q_1 & 0 & ----\\
0 & 0 & 0 & \tfrac{1}{2}Q_1 & \lambda_5 & x_5 & 0 & \tfrac{1}{2}Q_1 & ----\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{bmatrix}
\begin{Bmatrix}
x_{c1}\\ x_{s1}\\ x_{c3}\\ x_{s3}\\ x_{c5}\\ x_{s5}
\end{Bmatrix} = 0
\end{cases} \tag{51}$$

$$\underline{k = 1, 3, 5, --}$$

To obtain a solution, note that for $k \geq 3$ the equations can be written:

$$\underline{k \geq 3} \qquad \tfrac{1}{2}\begin{Bmatrix} Q_1 & 0 \\ 0 & Q_1 \end{Bmatrix}\left[\begin{Bmatrix} x_{c,k-2} \\ x_{s,k-2} \end{Bmatrix} + \begin{Bmatrix} x_{c,k+2} \\ x_{s,k+2} \end{Bmatrix}\right] + \begin{Bmatrix} x_k & -\lambda_k \\ \lambda_k & x_k \end{Bmatrix}\begin{Bmatrix} x_{ck} \\ x_{sk} \end{Bmatrix} = 0 \tag{52}$$

Set:

$$\begin{Bmatrix} x_{ck} \\ x_{sk} \end{Bmatrix} = \begin{Bmatrix} \alpha_{k-2} & -\beta_{k-2} \\ \beta_{k-2} & \alpha_{k-2} \end{Bmatrix}\begin{Bmatrix} x_{c,k-2} \\ x_{s,k-2} \end{Bmatrix} \tag{53}$$

and substitute into eq. (52) to get:

$$\alpha_{k-2} = -\frac{1}{2}Q_1 \frac{(\varkappa_k + \frac{1}{2}Q_1\alpha_k)}{(\varkappa_k + \frac{1}{2}Q_1\alpha_k)^2 + (\lambda_k + \frac{1}{2}Q_1\beta_k)^2} \tag{54}$$

$$\beta_{k-2} = \frac{1}{2}Q_1 \frac{(\lambda_k + \frac{1}{2}Q_1\beta_k)}{(\varkappa_k + \frac{1}{2}Q_1\alpha_k)^2 + (\lambda_k + \frac{1}{2}Q_1\beta_k)^2} \tag{55}$$

Noting the definition of $\varkappa_k$ and λ_k it is seen, that as $k \to \infty$, α_k and β_k go to zero. Hence, we may choose a sufficiently high value of k that the corresponding values of α_k and β_k can be set equal to zero without notably affecting the accuracy of the calculation. Starting from this value of k, and decreasing k in steps of 2, all the α_k's and β_k's can be computed from the recurrence relationships above, keeping the α_k's and β_k's for k even separated from the α_k's and β_k's for k odd. Carrying the calculations out to $k=4$ and $k=3$, respectively, eqs. (50) and (51) become:

$$\begin{Bmatrix} \begin{pmatrix} \varkappa_0 & \frac{1}{2}Q_1 & 0 \\ Q_1 & (\varkappa_2 + \frac{1}{2}Q_1\alpha_2) & -(\lambda_2 + \frac{1}{2}Q_1\beta_2) \\ 0 & (\lambda_2 + \frac{1}{2}Q_1\beta_2) & (\varkappa_2 + \frac{1}{2}Q_1\alpha_2) \end{pmatrix} \end{Bmatrix} \begin{Bmatrix} X_{c_0} \\ X_{c_2} \\ X_{s_2} \end{Bmatrix} = 0 \tag{56}$$

$$\begin{Bmatrix} \begin{pmatrix} (\varkappa_1 + \frac{1}{2}Q_1(\alpha_1+1)) & -(\lambda_1 + \frac{1}{2}Q_1\beta_1) \\ (\lambda_1 + \frac{1}{2}Q_1\beta_1) & (\varkappa_1 + \frac{1}{2}Q_1(\alpha_1-1)) \end{pmatrix} \end{Bmatrix} \begin{Bmatrix} X_{c_1} \\ X_{s_1} \end{Bmatrix} = 0 \tag{57}$$

When X_{c_k} and X_{s_k} are different from zero, the rotor is unstable. This requires that at least one of the determinants of the two matrices vanish. Hence, the zero-point of the two determinants establish the stability boundaries. To illustrate, assume there is no damping ($B=0$, i.e. $\lambda_k = 0$ and, therefore, $\beta_k = 0$). Furthermore, assume that Q_1 is sufficiently small that α_1 and α_2 can be ignored (note: α is "proportional" to Q_1, see eq. (54)). Under these assumptions the deter-

minants become:

$$\mathcal{X}_2\left[\mathcal{X}_0\,\mathcal{X}_2 - \tfrac{1}{2}\,Q_1^2\right] = 0 \tag{58}$$

$$(\mathcal{X}_1 + \tfrac{1}{2}\,Q_1)(\mathcal{X}_1 - \tfrac{1}{2}\,Q_1) = 0 \tag{59}$$

Substitute for $\mathcal{X}_0$, $\mathcal{X}_1$ and $\mathcal{X}_2$ from eq. (48) and introduce the critical speed of the rotor:

$$\omega_c^2 = \frac{K - Q_0}{m} \tag{60}$$

Then the determinants yield the solutions:

$$\underline{\mathcal{X}_2 = 0}: \qquad\qquad \frac{\Omega}{\omega_c} = 1 \tag{61}$$

$$\underline{\mathcal{X}_0\,\mathcal{X}_2 - \tfrac{1}{2}\,Q_1^2 = 0}: \qquad \left(\frac{Q_1}{K - Q_0}\right)^2 = 2\left[1 - \left(\frac{\Omega}{\omega_c}\right)^2\right] \tag{62}$$

$$\underline{(\mathcal{X}_1 + \tfrac{1}{2}\,Q_1)(\mathcal{X}_1 - \tfrac{1}{2}\,Q_1) = 0}: \qquad \frac{Q_1}{K - Q_0} = \pm\,2\left[1 - \tfrac{1}{4}\left(\frac{\Omega}{\omega_c}\right)^2\right] \tag{63}$$

These equations define the stability boundary for small values of $Q_1/(K - Q_0)$ and for $\mathcal{X}_3 < 0$, i.e. for $\frac{\Omega}{\omega_c} > \frac{2}{3}$. Graphically the equations can be shown on a stability map:
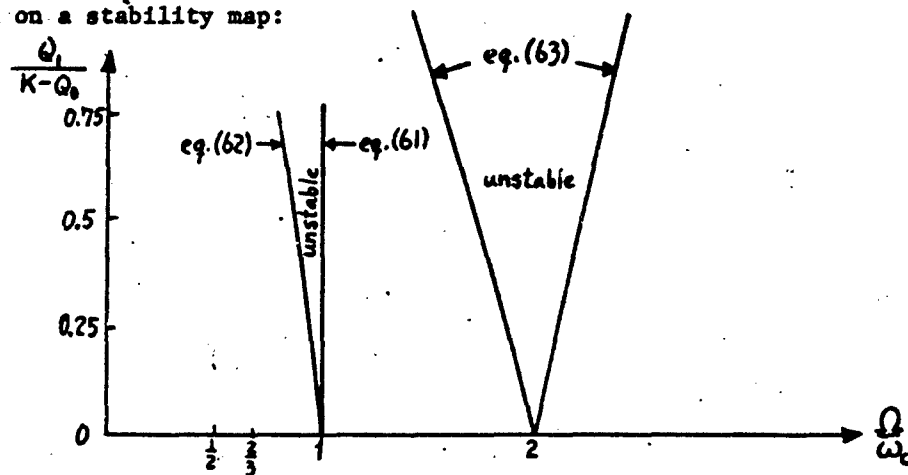
It is seen that the equations produce two zones of instability, one centered at $\frac{\Omega}{\omega_c} = 2$ and one at $\frac{\Omega}{\omega_c} = 1$. If more terms are carried in evaluating d_k, additional instability zones will be found, centered at $\frac{\Omega}{\omega_c} = \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, - - - $, but the zones become increasingly narrower. Furthermore, when damping is added the zones no longer reach $Q_1/(K-Q_0) = 0$. A more detailed plot is shown in Fig. 5 where the abcissa is Ω/ω_c, the ordinate is $Q_1/(K-Q_0)$ and there are curves showing the boundaries for different values of the damping parameter $B/2\sqrt{m(K-Q_0)}$ which gives the ratio between the damping coefficient B and the critical damping. The curves are calculated on the basis of the outlined analysis with a maximum value of k of 30.

A similar stability map is shown in Fig. 6 but whereas the first map is based on a timevarying magnetic force of the form $Q_1 \cos(\Omega t)$, the second map is based on a "square wave" variation:
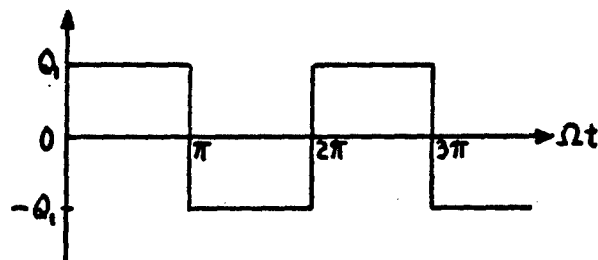
The difference between the two stability maps is not of any particular importance for most practical cases where $Q_1/(K-Q_0)$ seldom exceeds 1. The second map, however, illustrates that the changes in the stability zones caused by higher harmonics in the timevarying magnetic forces are small.

The analysis for calculating the stability map shown in Fig. 6 can be found in reference 1.

Turning next to the amplitude response of the rotor model considered above, assume that the rotor has a steady state eccentricity x_0' measured from the axis of the magnetic field, before the field is activated. When the magnetic field is activated, the rotor axis is pulled further out a distance x_0'' until a balance is reached between the rotor-bearing stiffness and the magnetic forces, i.e.

$$K x_0'' = Q_0 (x_0' + x_0'')$$

i.e.

$$x_0'' = \frac{Q_0}{K - Q_0} x_0' \tag{64}$$

or:

$$x_0 = x_0' + x_0'' = \frac{K}{K - Q_0} x_0' \tag{65}$$

Let the rotor amplitude x be measured from this equilibrium position. Hence, the equation of motion becomes:

$$m \frac{d^2 x}{dt^2} + B \frac{dx}{dt} + K(x_0'' + x) = (Q_0 - Q_1 \cos(\Omega t))(x_0 + x)$$

or:

$$m \frac{d^2 x}{dt^2} + B \frac{dx}{dt} + [K - Q_0 + Q_1 \cos(\Omega t)] x = - Q_1 x_0 \cos(\Omega t) \tag{66}$$

This equation is identical to eq. (42) except for the non-zero right hand side. Expand x in a Fourier series:

$$x = \sum_{k=0}^{\infty} [x_{c_k} \cos(k\psi) - x_{s_k} \sin(k\psi)] \tag{67}$$

where:

$$\psi = \Omega t \tag{68}$$

This expansion differs from the earlier one of eq. (43) by excluding all the terms with $\frac{1}{2}\Omega$ because these terms drop out in the response calculation. Hence, k is redefined and is half of its previous value. Except for this change, eq. (66) is expanded as shown earlier resulting in an infinite set of equations:

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$$\begin{Bmatrix} \varkappa_o & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & 0 & 0 & - & - \\ Q_1 & \varkappa_1 & -\lambda_1 & \tfrac{1}{2}Q_1 & 0 & 0 & 0 & - & - \\ 0 & \lambda_1 & \varkappa_1 & 0 & \tfrac{1}{2}Q_1 & 0 & 0 & - & - \\ 0 & \tfrac{1}{2}Q_1 & 0 & \varkappa_2 & -\lambda_2 & \tfrac{1}{2}Q_1 & 0 & - & - \\ 0 & 0 & \tfrac{1}{2}Q_1 & \lambda_2 & \varkappa_2 & 0 & \tfrac{1}{2}Q_1 & - & - \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \end{Bmatrix} \begin{Bmatrix} x_{co} \\ x_{c1} \\ x_{s1} \\ x_{c2} \\ x_{s2} \\ \vdots \\ \vdots \end{Bmatrix} = \begin{Bmatrix} 0 \\ -Q_1 x_o \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \end{Bmatrix} \qquad (69)$$

where:

$$\varkappa_k = K - Q_o - (k\Omega)^2 m \qquad (70)$$

$$\lambda_k = k\Omega B \qquad (71)$$

These equations are reduced to 3 equations by the procedure employed previously to:

$$\begin{Bmatrix} \varkappa_o & \tfrac{1}{2}Q_1 & 0 \\ Q_1 & (\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1) & -(\lambda_1 + \tfrac{1}{2}Q_1\beta_1) \\ 0 & (\lambda_1 + \tfrac{1}{2}Q_1\beta_1) & (\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1) \end{Bmatrix} \begin{Bmatrix} x_{co} \\ x_{c1} \\ x_{s1} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -Q_1 x_o \\ 0 \end{Bmatrix} \qquad (72)$$

The equations are readily solved to give:

$$x_{co} = \frac{\tfrac{1}{2}Q_1^2(\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1)}{\varkappa_o[(\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1)^2 + (\lambda_1 + \tfrac{1}{2}Q_1\beta_1)^2] - \tfrac{1}{2}Q_1^2(\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1)} \cdot x_o \qquad (73)$$

$$x_{c1} = - \frac{\varkappa_o Q_1(\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1)}{\varkappa_o[(\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1)^2 + (\lambda_1 + \tfrac{1}{2}Q_1\beta_1)^2] - \tfrac{1}{2}Q_1^2(\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1)} \cdot x_o \qquad (74)$$

$$x_{s1} = - \frac{\lambda_1 + \tfrac{1}{2}Q_1\beta_1}{\varkappa_1 + \tfrac{1}{2}Q_1\alpha_1} \cdot x_{c1} \qquad (75)$$

For simplification, assume that there is no damping ($B = 0$, i.e. $\lambda_k = 0$ and, therefore, $\beta_k = 0$). Furthermore, assume that α_1 can be ignored. Introducing the critical speed ω_c from eq. (60) the above equations become:

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$$X_{c0} \cong \frac{\frac{1}{2}\left(\frac{Q_1}{K-Q_0}\right)^2}{1-\left(\frac{\Omega}{\omega_c}\right)^2 - \frac{1}{2}\left(\frac{Q_1}{K-Q_0}\right)^2} \cdot X_0 \tag{76}$$

$$X_{c1} \cong -\frac{\frac{Q_1}{K-Q_0}}{1-\left(\frac{\Omega}{\omega_c}\right)^2 - \frac{1}{2}\left(\frac{Q_1}{K-Q_0}\right)^2} \cdot X_0 \tag{77}$$

$$X_{s1} \cong 0 \tag{78}$$

X_{c0} gives the shift in equilibrium position beyond the previously determined X_0^n, see eq. (64), such that the total static eccentricity between the magnetic axis and the rotor axis is:

$$X_0 + X_{c0} \cong \frac{1-\left(\frac{\Omega}{\omega_c}\right)^2}{1-\left(\frac{\Omega}{\omega_c}\right)^2 - \frac{1}{2}\left(\frac{Q_1}{K-Q_0}\right)^2} \cdot \frac{K}{K-Q_0} \cdot X_0' \tag{79}$$

where X_0' is the mechanically built-in eccentricity.

Disregarding the frequently small term $\frac{1}{2}\left(\frac{Q_1}{K-Q_0}\right)^2$, the amplitude X_{c1} is the same as would be obtained if the term $Q_1 \cos(\Omega t)$ was ignored on the left hand side of eq. (66). Hence, under the stated condition the analysis can be simplified significantly.

In the preceeding it has been assumed that the rotor is rigid and symmetric and, furthermore, that only one amplitude direction needs to be considered. Also, such a simple rotor model gives rise to only one resonance ("critical speed"). Although these assumptions are reasonably valid in some applications, many rotors are unsymmetric or flexible and all rotors have more than one critical speed. In addition, the timevarying magnetic forces, and frequently also the bearing stiffness and damping, cause coupling between the x and y amplitude directions. Hence, a much more extensive analysis is required to treat an arbitrary rotor. However, the

basic principle of the preceeding analysis is still valid. Returning to eq. (46), which forms the basis of the previous solution, it can be written:

$$\sum_{k=0}^{\infty}\left[(\varkappa_k x_{ck}-\lambda_k x_{sk})\cos(k\psi)-(\lambda_k x_{ck}+\varkappa_k x_{sk})\sin(k\psi)\right]+Q_1\cos(2\psi)\sum_{k=0}^{\infty}\left[x_{ck}\cos(k\psi)-x_{sk}\sin(k\psi)\right]=0 \quad (80)$$

where $\varkappa_k$ and λ_k are given by eqs. (48) and (49). Introduce a complex notation:

$$x_k = x_{ck} + i\,x_{sk} \quad (81)$$

which is actually an abbreviated notation which in its complete form reads:

$$x_k = Re\left\{(x_{ck}+ix_{sk})e^{i\psi}\right\} = x_{ck}\cos(k\psi)-x_{sk}\sin(k\psi) \quad (82)$$

where $Re\{\ \ \}$ means that only the real part of the bracketed expression applies. For convenience, both $Re\{\ \ \}$ and $e^{i\psi}$ are dropped during the detailed analysis and only brought back in the final answer. With this convention, eq. (80) can be written:

$$\sum_{k=0}^{\infty}(\varkappa_k+i\lambda_k)x_k + Q_1\cos(2\psi)\sum_{k=0}^{\infty}x_k = 0 \quad (83)$$

($\varkappa_k + i\lambda_k$) is called the impedance of the rotor. It gives the ratio between an applied force F_{xk} with a given frequency and the resulting amplitude x_k. This is seen by simply applying a force F_{xk} with a frequency $\left(k\frac{\Omega}{2}\right)$, to the rotor alone without timevarying magnetic forces. The equation of motion becomes (see eq. (42)):

$$m\frac{d^2x_k}{dt^2} + B\frac{dx_k}{dt} + (K-Q_0)x_k = F_{xk}$$

where:

$$F_{xk} = F_{xck}+i\,F_{ysk} = F_{xck}\cos(k\psi) - F_{xsk}\sin(k\psi)$$

Then the solution is:

$$\frac{F_{xk}}{x_k} = \left[K-Q_0-\left(k\frac{\Omega}{2}\right)^2 m\right] + i\,k\frac{\Omega}{2}B = \varkappa_k+i\lambda_k \quad (84)$$

which shows the meaning of the impedance ($\varkappa_k + i\lambda_k$).

The response at any location on an arbitrary rotor can also be represented by impedances, and these impedances are determined by applying known dynamic forces at the particular location on the rotor, computing the corresponding amplitude and taking the ratio. In an arbitrary rotor it is necessary to apply forces in both the x and y-directions so that in total:

$$F_{xk} = (\varkappa_{11} + i\lambda_{11})_k x_k + (\varkappa_{12} + i\lambda_{12})_k y_k$$

(85)

$$F_{yk} = (\varkappa_{21} + i\lambda_{21})_k x_k + (\varkappa_{22} + i\lambda_{22})_k y_k$$

Letting the frequency $\left(k\frac{\Omega}{2}\right)$ of the applied forces take on all desired values, the impedances can be determined for $k = 0, 1, 2, ---$. These impedances can then be substituted into equations, one for the x-direction and one for the y-direction, of the same general form as eq. (46). Thereafter the corresponding infinite matrices can be formed, analogous to eqs. (50) and (51), and used in either a stability investigation or a response calculation as discussed previously.

DISCUSSION ON PERFORMING STABILITY CALCULATIONS

The general method employed in calculating the threshold of instability is discussed in the preceeding chapter and the detailed analysis is given in Appendix IX In order to make proper use of the stability computer program, described in details in Appendix XI it is necessary to be rather familiar with the analysis. Therefore, a brief discussion will be given in the following to illustrate how some knowledge of the analysis is essential to a proper interpretation of the computer output.

It should first be remarked that the computer program is written for an arbitrary rotor where, as an example, it is assumed that the bearings have different stiffnesses and damping in the vertical and horizontal directions and, furthermore, that there is coupling between the motion in the two directions. Some rotor-bearing systems are axisymmetric in which case the motions in the two directions are the same and the program does twice as many calculations as are actually necessary. Then the stability determinants tend to stay positive and the threshold of instability is not readily found because of the difficulties involved in determining those particular points where the determinants may assume a value of zero. Hence, it can be said that the program's ability to handle the completely general case sometimes makes it more difficult to examine simpler systems.

The discussion is best carried out by taking an example for illustration. The two instability determinants are given by eqs. (J.25) and (J.28), Appendix IX,as:

$$\left| E_{c_0} + \tfrac{1}{2} Q S_{c_0} + \tfrac{1}{2} q S_{s_0} \right| = 0 \qquad \text{(even indices)} \qquad (86)$$

$$\text{Determinant of } \left\{ (E_1 + H S_1) X_1 + \tfrac{1}{2} Q X_{c_1} + \tfrac{1}{2} q X_{s_1} + i \left(\tfrac{1}{2} q X_{c_1} - \tfrac{1}{2} Q X_{s_1} \right) \right\} = 0 \qquad (87)$$
$$\text{(odd indices)}$$

where reference is made to Appendix IX for the meaning of the symbols. Consider a four pole homopolar generator for which the magnetic force gradients yield the two matrices (see Appendix I):

$$Q = \left\{ \begin{array}{cccc} 0 & 0 & 2a & 0 \\ 0 & 0 & 0 & -2a \\ 2a & 0 & 0 & 0 \\ 0 & -2a & 0 & 0 \end{array} \right\} \tag{88}$$

$$q = \left\{ \begin{array}{cccc} 0 & 0 & 0 & -2a \\ 0 & 0 & -2a & 0 \\ 0 & -2a & 0 & 0 \\ -2a & 0 & 0 & 0 \end{array} \right\} \tag{89}$$

where:

$$a = -\frac{A B_o^2}{72 C} L_p \tag{90}$$

Let the rotor be symmetric and such that there is no coupling between the x-amplitudes and the y-amplitudes. Hence, the rotor impedance matrices, E_k, can be written (eq. (H.72), Appendix VIII):

$$E_k = \left\{ \begin{array}{cccc} (\varkappa_x + i\lambda_x)_k & 0 & 0 & 0 \\ 0 & (\varkappa_y + i\lambda_y)_k & 0 & 0 \\ 0 & 0 & (\varkappa_\theta + i\lambda_\theta)_k & 0 \\ 0 & 0 & 0 & (\varkappa_\varphi + i\lambda_\varphi)_k \end{array} \right\} \tag{91}$$

If the rotor is rigid with a mass m, a transverse mass moment of inertia I and a negligible polar mass moment of inertia, the bearing stiffnesses are K_x and K_y, and the bearing dampings are B_x and B_y, the elements of the impedance matrix become:

$$\varkappa_{xk} = 2 K_x - (k\nu)^2 m$$
$$\varkappa_{yk} = 2 K_y - (k\nu)^2 m$$
$$\varkappa_{\theta k} = \tfrac{1}{2} K_x \ell^2 - (k\nu)^2 I$$
$$\varkappa_{\varphi k} = \tfrac{1}{2} K_y \ell^2 - (k\nu)^2 I$$
$$\lambda_{xk} = 2 (k\nu) B_x \tag{92}$$
$$\lambda_{yk} = 2 (k\nu) B_y$$
$$\lambda_{\theta k} = \tfrac{1}{2} \ell^2 (k\nu) B_x$$
$$\lambda_{\varphi k} = \tfrac{1}{2} \ell^2 (k\nu) B_y$$

where l is the rotor span between bearings and $\nu = \frac{1}{2}\Omega$, where Ω is the frequency of the magnetic forces.

Consider eq. (87) and restrict the analysis to a single harmonic only (i.e. $S_i = 0$). Substitute from eqs. (88), (89) and (91) to get:

$$
\begin{Bmatrix}
\varkappa_{x1} & -\lambda_{x1} & 0 & 0 & a & 0 & 0 & -a \\
\lambda_{x1} & \varkappa_{x1} & 0 & 0 & 0 & -a & -a & 0 \\
0 & 0 & \varkappa_{y1} & -\lambda_{y1} & 0 & -a & -a & 0 \\
0 & 0 & \lambda_{y1} & \varkappa_{y1} & -a & 0 & 0 & a \\
a & 0 & 0 & -a & \varkappa_{\theta1} & -\lambda_{\theta1} & 0 & 0 \\
0 & -a & -a & 0 & \lambda_{\theta1} & \varkappa_{\theta1} & 0 & 0 \\
0 & -a & -a & 0 & 0 & 0 & \varkappa_{\varphi1} & -\lambda_{\varphi1} \\
-a & 0 & 0 & a & 0 & 0 & \lambda_{\varphi1} & \varkappa_{\varphi1}
\end{Bmatrix}
\begin{Bmatrix}
x_{c1} \\ x_{s1} \\ y_{c1} \\ y_{s1} \\ \theta_{c1} \\ \theta_{s1} \\ \varphi_{c1} \\ \varphi_{s1}
\end{Bmatrix} = 0
\qquad (93)
$$

The determinant of this matrix is the determinant for odd indices. To evaluate the determinant it is seen that the systems of equations can be written as two sets of equations:

$$
\begin{Bmatrix}
(\varkappa_{x1}+i\lambda_{x1}) & \\
0 & (\varkappa_{y1}+i\lambda_{y1})
\end{Bmatrix}
\begin{Bmatrix}
x_{c1}+ix_{s1} \\ y_{c1}+iy_{s1}
\end{Bmatrix}
+
\begin{Bmatrix}
a & -ia \\ -ia & -a
\end{Bmatrix}
\begin{Bmatrix}
\theta_{c1}-i\theta_{s1} \\ \varphi_{c1}-i\varphi_{s1}
\end{Bmatrix} = 0
\qquad (94)
$$

$$
\begin{Bmatrix}
a & ia \\ ia & -a
\end{Bmatrix}
\begin{Bmatrix}
x_{c1}+ix_{s1} \\ y_{c1}+iy_{s1}
\end{Bmatrix}
+
\begin{Bmatrix}
(\varkappa_{\theta1}-i\lambda_{\theta1}) & 0 \\ 0 & (\varkappa_{\varphi1}-i\lambda_{\varphi1})
\end{Bmatrix}
\begin{Bmatrix}
\theta_{c1}-i\theta_{s1} \\ \varphi_{c1}-i\varphi_{s1}
\end{Bmatrix} = 0
\qquad (95)
$$

Solve eq. (95) for $\begin{Bmatrix} \theta_{c1}-i\theta_{s1} \\ \varphi_{c1}-i\varphi_{s1} \end{Bmatrix}$ and substitute into eq. (94) to get:

$$
\left[
\begin{Bmatrix}
(\varkappa_{x1}+i\lambda_{x1}) & 0 \\ 0 & (\varkappa_{y1}+i\lambda_{y1})
\end{Bmatrix}
-
\begin{Bmatrix}
a & -ia \\ -ia & -a
\end{Bmatrix}
\begin{Bmatrix}
1/(\varkappa_{\theta1}-i\lambda_{\theta1}) & 0 \\ 0 & 1/(\varkappa_{\varphi1}-i\lambda_{\varphi1})
\end{Bmatrix}
\begin{Bmatrix}
a & ia \\ ia & -a
\end{Bmatrix}
\right]
\begin{Bmatrix}
x_{c1}+ix_{s1} \\ y_{c1}+iy_{s1}
\end{Bmatrix} = 0
\qquad (96)
$$

or:

$$
\begin{Bmatrix}
(\varkappa_{x1}-\xi+i\lambda_{x1}) & -i\xi \\
i\xi & (\varkappa_{y1}-\xi+i\lambda_{y1})
\end{Bmatrix}
\begin{Bmatrix}
x_{c1}+ix_{s1} \\ y_{c1}+iy_{s1}
\end{Bmatrix} = 0
\qquad (97)
$$

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where:

$$\xi = a^2\left[\frac{1}{\varkappa_{\theta_1}-i\lambda_{\theta_1}} + \frac{1}{\varkappa_{\varphi_1}+i\lambda_{\varphi_1}}\right] = a^2\left[\frac{\varkappa_{\theta_1}+i\lambda_{\theta_1}}{\varkappa_{\theta_1}^2+\lambda_{\theta_1}^2} + \frac{\varkappa_{\varphi_1}+i\lambda_{\varphi_1}}{\varkappa_{\varphi_1}^2+\lambda_{\varphi_1}^2}\right] \tag{98}$$

The determinant becomes:

$$\Delta_{odd} = (\varkappa_{x_1}-\xi+i\lambda_{x_1})(\varkappa_{y_1}-\xi+i\lambda_{y_1})-\xi^2 = (\varkappa_{x_1}+i\lambda_{x_1})(\varkappa_{y_1}+i\lambda_{y_1})-\xi(\varkappa_{x_1}+i\lambda_{x_1}+\varkappa_{y_1}+i\lambda_{y_1}) \tag{99}$$

which is zero for:

$$\xi = \left[\frac{1}{\varkappa_{x_1}+i\lambda_{x_1}} + \frac{1}{\varkappa_{y_1}+i\lambda_{y_1}}\right]^{-1} \tag{100}$$

or for:

$$a^2 = \left[\frac{1}{\varkappa_{x_1}+i\lambda_{x_1}} + \frac{1}{\varkappa_{y_1}+i\lambda_{y_1}}\right]^{-1}\left[\frac{1}{\varkappa_{\theta_1}-i\lambda_{\theta_1}} + \frac{1}{\varkappa_{\varphi_1}-i\lambda_{\varphi_1}}\right]^{-1} \tag{101}$$

Assume that the bearings have no damping, i.e. $\lambda_{x_1}=\lambda_{y_1}=\lambda_{\theta_1}=\lambda_{\varphi_1}=0$, whereby eq. (101) becomes:

$$a^2 = \frac{\varkappa_{x_1}\varkappa_{y_1}\varkappa_{\theta_1}\varkappa_{\varphi_1}}{(\varkappa_{x_1}+\varkappa_{y_1})(\varkappa_{\theta_1}+\varkappa_{\varphi_1})} \tag{102}$$

The rotor has four critical speeds:

$$\omega_x = \sqrt{\frac{2K_x}{m}}$$

$$\omega_y = \sqrt{\frac{2K_y}{m}} \tag{103}$$

$$\omega_\theta = \sqrt{\frac{\frac{1}{2}\ell^2 K_x}{m}}$$

$$\omega_\varphi = \sqrt{\frac{\frac{1}{2}\ell^2 K_y}{m}}$$

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Substitute from eqs. (92) into eq. (102), making use of eqs. (103):

$$\frac{a^2}{l^2 K_x K_y} = \frac{\left[1-\left(\frac{\nu}{\omega_x}\right)^2\right]\left[1-\left(\frac{\nu}{\omega_y}\right)^2\right]\left[1-\left(\frac{\nu}{\omega_\theta}\right)^2\right]\left[1-\left(\frac{\nu}{\omega_\varphi}\right)^2\right]}{\left[1+\left(\frac{\omega_x}{\omega_y}\right)^2-2\left(\frac{\nu}{\omega_y}\right)^2\right]\left[1+\left(\frac{\omega_\theta}{\omega_\varphi}\right)^2-2\left(\frac{\nu}{\omega_\varphi}\right)^2\right]} \qquad (104)$$

Since it is a four pole homopolar generator, $\nu = \omega$ where ω is the angular speed of the rotor.

The stability map defined by eq. (104) is best illustrated by assuming certain numerical values. Let $K_y = \frac{1}{2} K_x$ (i.e. the bearings are twice as stiff in the x-direction as in the y-direction). Then:

$$\left(\frac{\omega_x}{\omega_y}\right)^2 = \left(\frac{\omega_\theta}{\omega_\varphi}\right)^2 = \frac{K_x}{K_y} = 2$$

Assume also, that the conical critical speed, ω_θ, is 1.58 times the translatory critical speed ω_x such that:

$$\left(\frac{\omega_\theta}{\omega_x}\right)^2 = 2.5$$

Then:

$$\frac{a^2}{l^2 K_x K_y} = \frac{\left[1-\left(\frac{\omega}{\omega_x}\right)^2\right]\left[1-2\left(\frac{\omega}{\omega_x}\right)^2\right]\left[1-0.4\left(\frac{\omega}{\omega_x}\right)^2\right]\left[1-0.8\left(\frac{\omega}{\omega_x}\right)^2\right]}{9\left[1-\frac{4}{3}\left(\frac{\omega}{\omega_x}\right)^2\right]\left[1-\frac{1.6}{3}\left(\frac{\omega}{\omega_x}\right)^2\right]}$$

The corresponding stability plot becomes:

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$$\frac{a}{2\sqrt{K_x K_y}}$$

Unstable   Unstable

0.5 0.4 0.3 0.2 0.1 0

0.5 $\frac{1}{\sqrt{2}}$ $\frac{\sqrt{3}}{2}$ 1 $\sqrt{1.25}$ $\frac{\sqrt{3}}{\sqrt{1.4}}$ $\sqrt{2.5}$ $\frac{\omega}{\omega_x}$
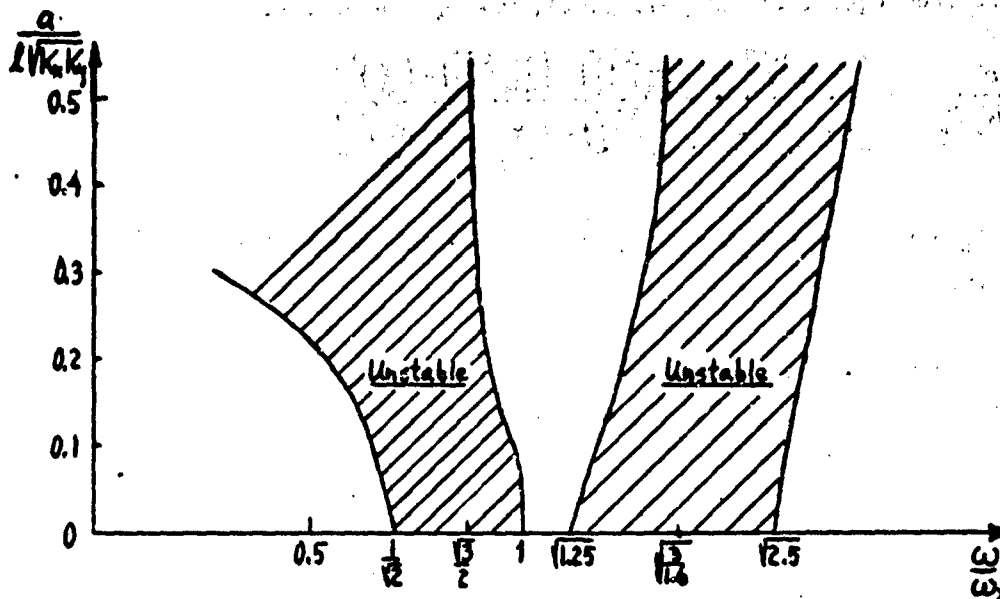
When it is now considered that there will be analogous instability zones located at $\frac{\omega}{\omega_x} = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, etc. and, furthermore, for $\frac{\omega}{\omega_x} > 1$ there may be instability zones caused by the higher critical speeds of the rotor, it is readily seen that the complete stability map can easily become very complicated. However, it is necessary to have some preconcept of where the instability zones are, otherwise it is easy to misinterpret the results from the computer program. Thus, if in the above example the rotor is operating at $\frac{\omega}{\omega_x} = 1.5$, the stability determinant would never be equal to zero because the rotor is inherently unstable and the determinant only indicates the threshold of instability, i.e. the boundaries of the instability zones. It cannot tell if the rotor is stable or unstable.

When bearing damping is present, the instability zones move up in the above map and some of the zones may actually disappear altogether. It is still recommended to perform a calculation without damping first in order to locate the potential instability zones. It is then easier to decide where to search for the instability threshold when damping is included.

The computer program searches for the threshold of instability by varying
the value of the magnetic force gradient. In other words, in terms of the
above example, a is varied over a specified range at a fixed value of ω/ω_x
and for each value of a the two determinants are computed. The program detects
if any of the determinants changes sign but is, of course, otherwise unable to
decide if a determinant has a zero-point. The numerical round-off errors and
also the fact that the determinants are only evaluated at discrete values,
usually prevent the detection of a zero-point where the slope of the determinant
is zero. For this reason it is frequently necessary to let the magnetic force
gradient vary in fine increments.

In summary, the recommended procedure for performing a stability calculation is:

<u>a</u>. Determine all the critical speeds of the rotor that may possibly influence
the stability of the rotor. These are the critical speeds which are close to
1/2, 1 and maybe even 3/2 times the frequency of the magnetic forces (it
depends on how well they are damped). The two lowest critical speeds should
always be included and frequently also the third critical speed.

<u>b</u>. The magnetic force frequency, Ω , is a fixed ratio of the rotor speed ω .
For the four pole homopolar generator, $\frac{\Omega}{\omega} = 2$ and for the heteropolar inductor
generator, $\frac{\Omega}{\omega}$ is equal to the number of rotor teeth. Then, on the basis
of the known critical speeds select two rotor speeds, one on each side of the
operating speed. These two speeds are determined as the speeds closest to the
operating speed from the following relationships:

$$\omega = \begin{Bmatrix} 2 \\ 1 \\ \frac{2}{3} \end{Bmatrix} \frac{\omega_{critical}}{\left(\frac{\Omega}{\omega}\right)}$$

To illustrate, assume that a four pole homopolar generator operates at 12,000 rpm
($\frac{\Omega}{\omega} = 2$) and that its first three critical speeds are at 9,000, 11,000 and
32,000 rpm. Then the rotor speeds at which the rotor is susceptible to instability
are:

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9,000 rpm	11,000 rpm	32,000 rpm
4,500 rpm	5,500 rpm	16,000 rpm
3,000 rpm	3,700 rpm	10,700 rpm

Hence, the minimum speed range for the calculations are from 11,000 to 16,000 rpm.

c. Perform stability calculations covering the determined speed range and leaving
out any bearing damping. In this way, a stability map for the undamped system
is obtained. If the actual magnetic force gradients are such that the rotor
operates in a stable zone, the rotor is stable and no further calculations
are required. Otherwise, perform additional calculations in which the bearing
damping is included. In these calculations the magnetic force gradients should
be varied in very small steps in the neighborhood of the threshold in order to
determine the exact zero-point (or minimum point) of the instability determinant.
If the rotor operates below the zero-point it is stable, otherwise unstable
(in theory there are exceptions to this rule but in practice the rule should be
valid).

DISCUSSION ON PERFORMING RESPONSE CALCULATIONS

A rotor response calculation is considerably simpler to perform than a stability calculation and does not require the same understanding of the detailed analysis. However, some knowledge of the analysis may prove helpful in certain cases.

Let the frequency of the magnetic forces be Ω and the angular speed of the rotor is ω. The ratio: Ω/ω is fixed for a given generator ($\frac{\Omega}{\omega} = 2$ for the 4 pole homopolar generator, and $\frac{\Omega}{\omega}$ is equal to the number of teeth for the heteropolar inductor generator). The rotor is forced to whirl by the magnetic forces produced when the rotor axis does not coincide with the magnetic axis of the alternator stator. The position of the rotor axis is defined by four coordinates: the eccentricity components X_0 and y_0 measured in the center-plane of the alternator, and the misalignment angles Θ_0 and φ_0. In the homopolar generator, both forces and moments will be set up such that the forces are proportional to Θ_0 and φ_0 and the moments are proportional to X_0 and y_0. In the heteropolar inductor generator, only forces are produced. They are proportional to X_0 and y_0.

The fundamental response of the rotor has the same frequency as the magnetic forces (i.e. the amplitudes vary harmonically with the frequency Ω). In addition, higher harmonics of the fundamental frequency will also be excited which means that the vibratory response will contain components not only with the fundamental frequency Ω but also with frequencies 2Ω, 3Ω, and so on. However, the excitation force available for the higher harmonics normally decrease rapidly with the number of the harmonics. Let the gradients of the magnetic forces be represented by the symbol a and let the combined rotor-bearing stiffness be represented by K. If the number of the harmonic is n, the available excitation force for that harmonic is very roughly proportional to $\left(\frac{a}{K}\right)^{(2n-1)}$. Thus, if the magnetic force gradient is, say 30 percent of the rotor-bearing stiffness (i.e. $\frac{a}{K} = 0.3$) and resonance effects are ignored, the amplitudes of the second harmonic are of the order of 10 percent of the amplitudes of the fundamental harmonic, and the amplitudes of the third harmonic are only of the order of 1 percent of the fundamental harmonic. Hence, it is readily seen that

unless $\frac{a}{K}$ is reasonably large, only the fundamental harmonic, or possibly the two first harmonics, are of any practical significance. The only possible exception is when one of the harmonic frequencies is close to a resonance of the rotor-bearing system for which little damping is provided. In that case, even if the excitation force may be small, the corresponding amplitudes could become appreciable. The resonant peak, on the other hand, will be very narrow. It is, therefore, recommended that when a rotor response calculation is performed, the calculation is not limited just to the operating speed but covers a reasonable speed range around the operating speed. In this way it will be possible to detect if there are any high amplitude response too close to the operating speed.

SUMMARY

The principal objectives of this volume are: a) to give formulas from which
the magnetic forces in three representative generator types can be calculated,
b) to provide a computer program to calculate the stability of the alternator
rotor, and c) to provide a computer program to calculate the amplitude response
of the alternator rotor. It is the intention that these engineering tools can
be used in future design and development work in the application of alternators
to space power plants and similar machinery, and both the formulas and the two
computer programs are presented in a form where they can be readily applied
to an actual application.

To establish the formulas and the computational methods, a rather complex analysis
has been performed. There is no previous work in this field on which the
analysis can be based and it is believed that several of the developed methods
may be of value in future work on electromagnetic force interaction and rotor
dynamics.

At present there is little test experience or experimental data against which the
results of this investigation can be compared. For this reason and, more sig-
nificantly, also because it is a problem of serious practical concern, it would
be desirable to perform a similar investigation of the effect of the magnetic
forces on the rotor of an electrical motor. As mentioned previously, severe
vibration problems have been encountered in at least three electrical motor
applications and it would be of importance to determine the exact causes of
the vibrations so that the problem may be avoided in future motor designs.
The methods presented in this volume could serve as a basis for such an
investigation.

ACKNOWLEDGMENT

FIGURES

47
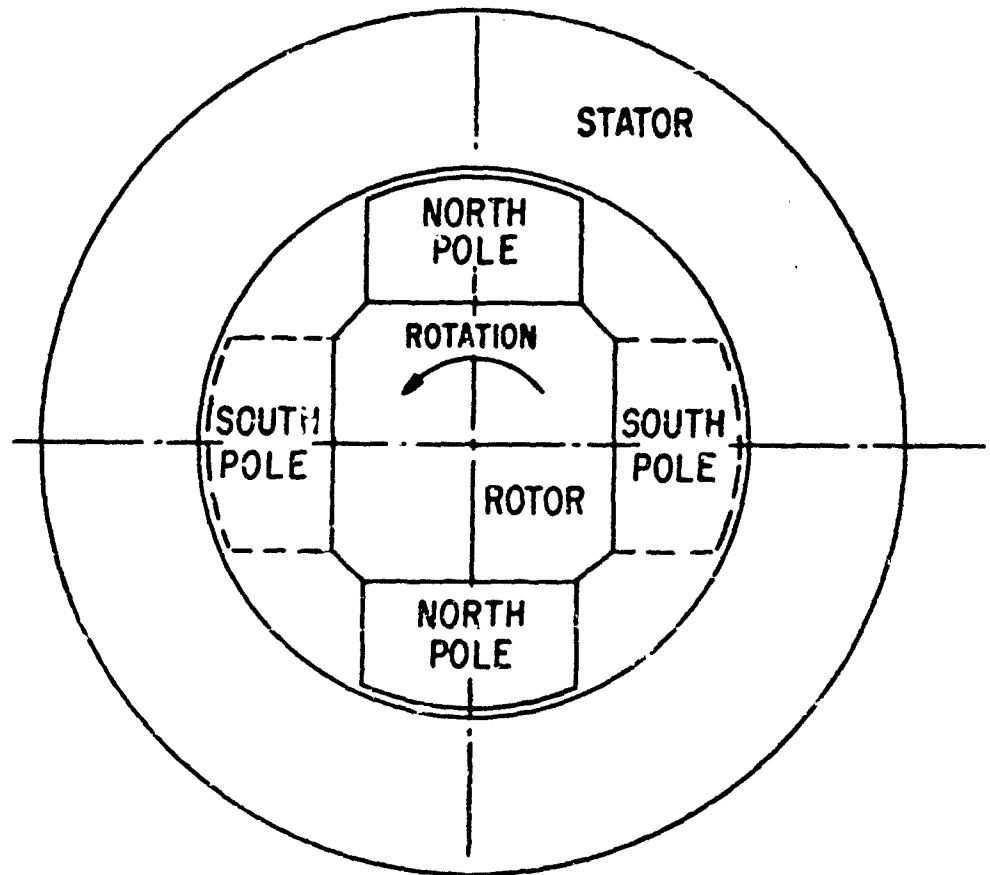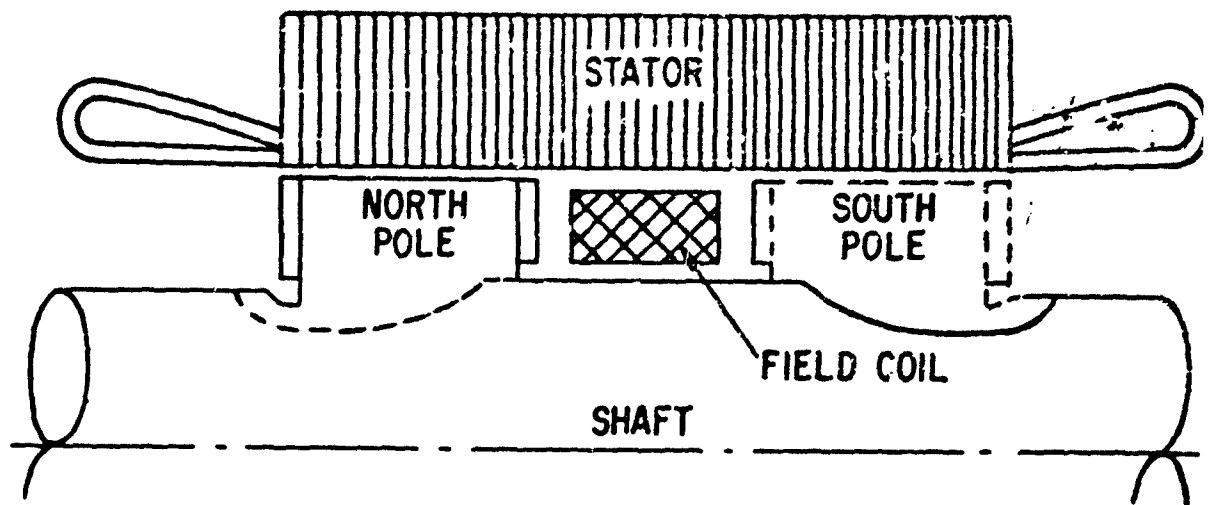
Figure 1   Homopolar Generator

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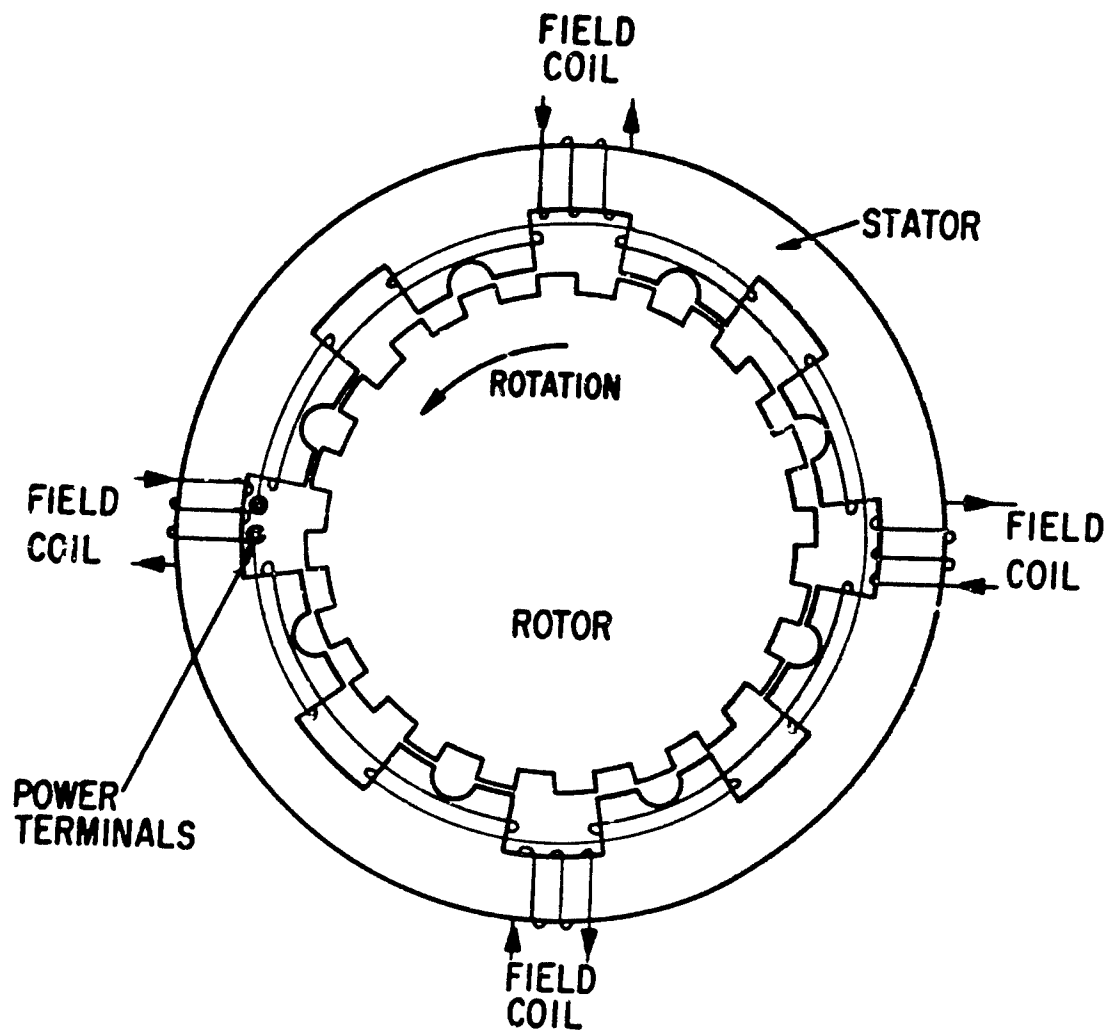
Figure 2   Heteropolar Generator

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STATOR

SOUTH POLE   SOUTH POLE

NORTH
POLE

FIELD
COIL

ROTOR

SHAFT
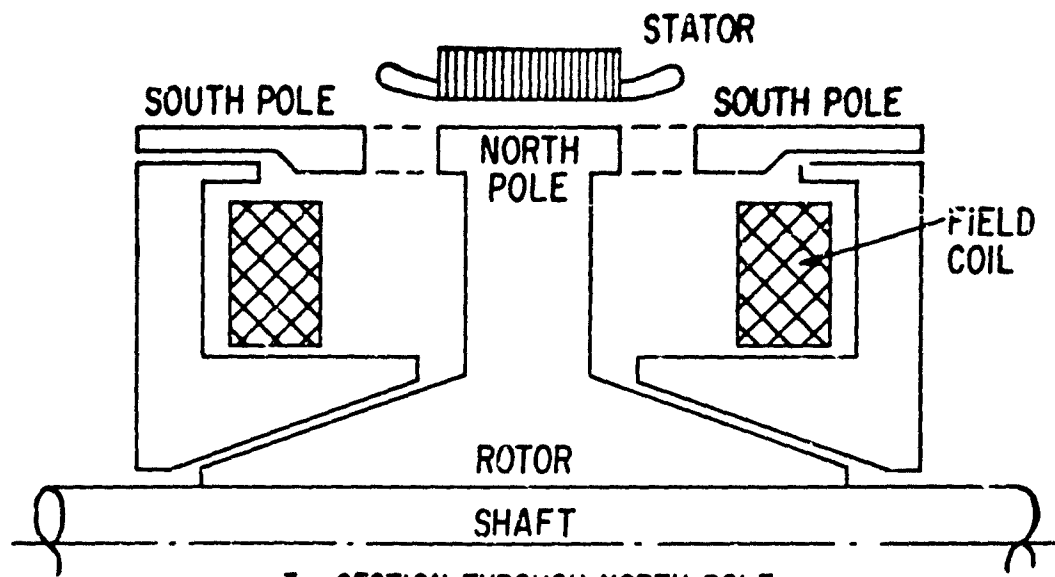
3a SECTION THROUGH NORTH POLE

C

L_1

R

GAP "g_3"   SOUTH POLE   GAP "g_1"

GAP "g_4"   GAP "g_2"

L_2

C   R
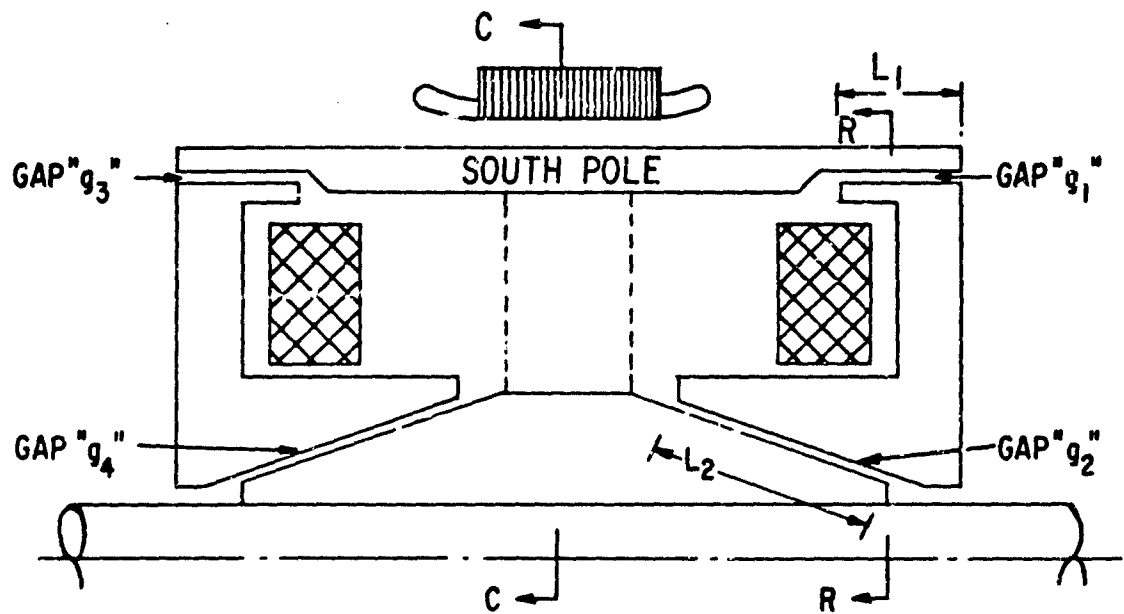
3b SECTION THROUGH SOUTH POLE

Figure 3   Two-Coil Lundell Generator

MTI-3896
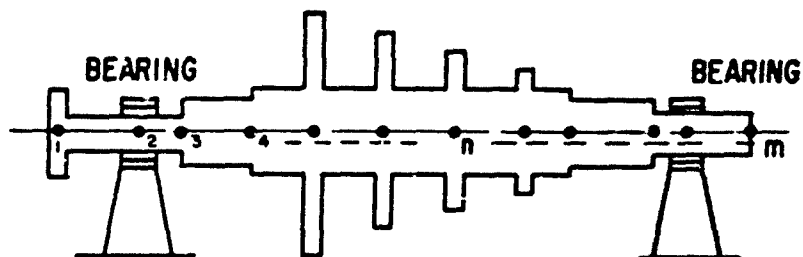
Outline of Rotor with Location of Rotor Stations
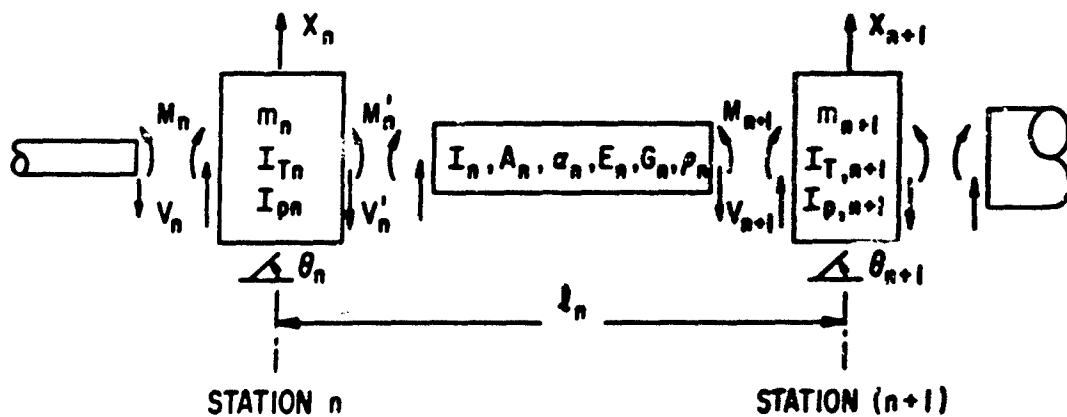
Sign Convention for Amplitude, Slope, Bending Moment and Shear Force

Figure 4   Rotor Model and Sign Convention for Analysis of Rotor Impedance
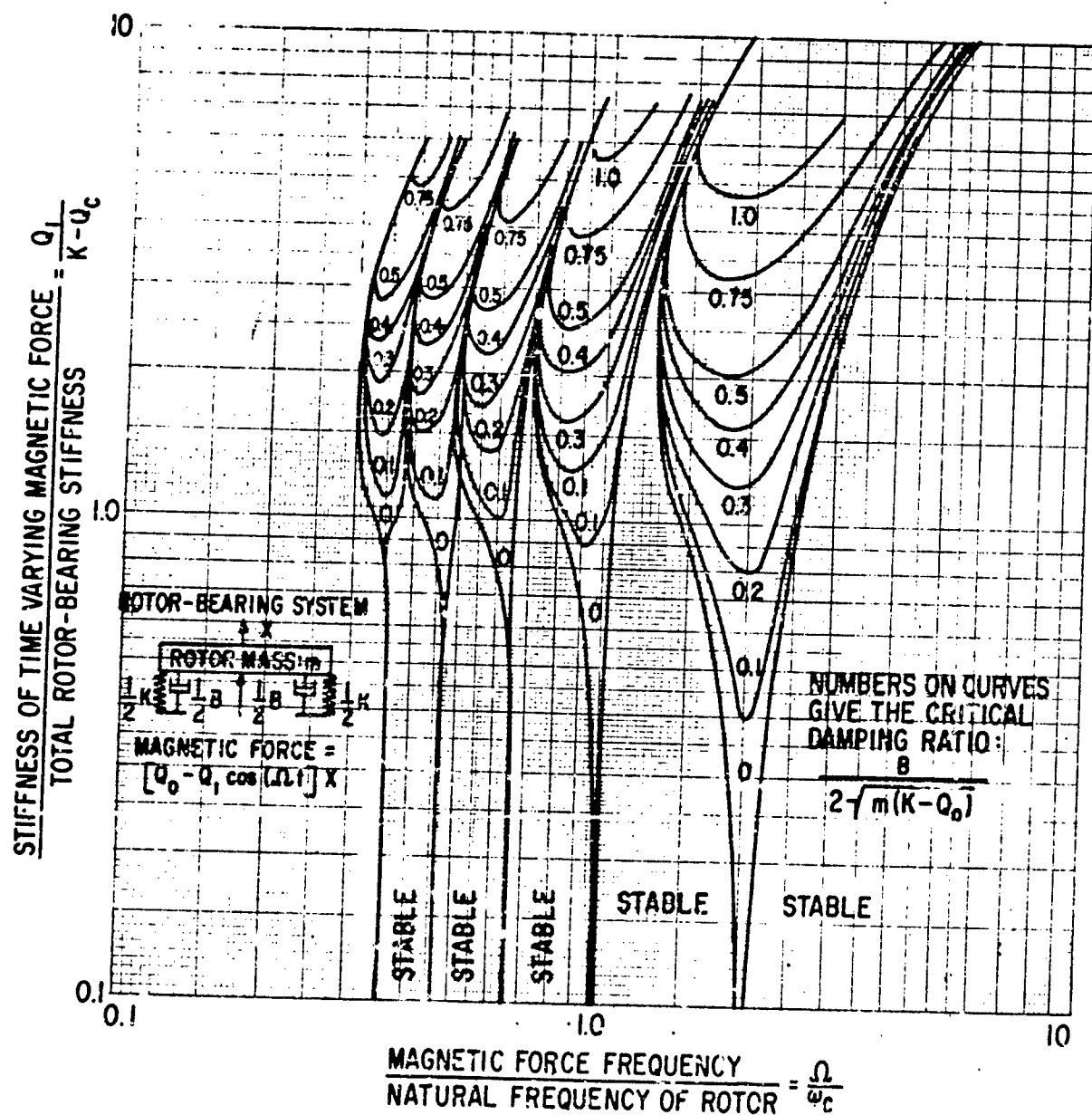
Figure 5   Stability Map For A Rigid Rotor With A Harmonically
Varying Magnetic Force
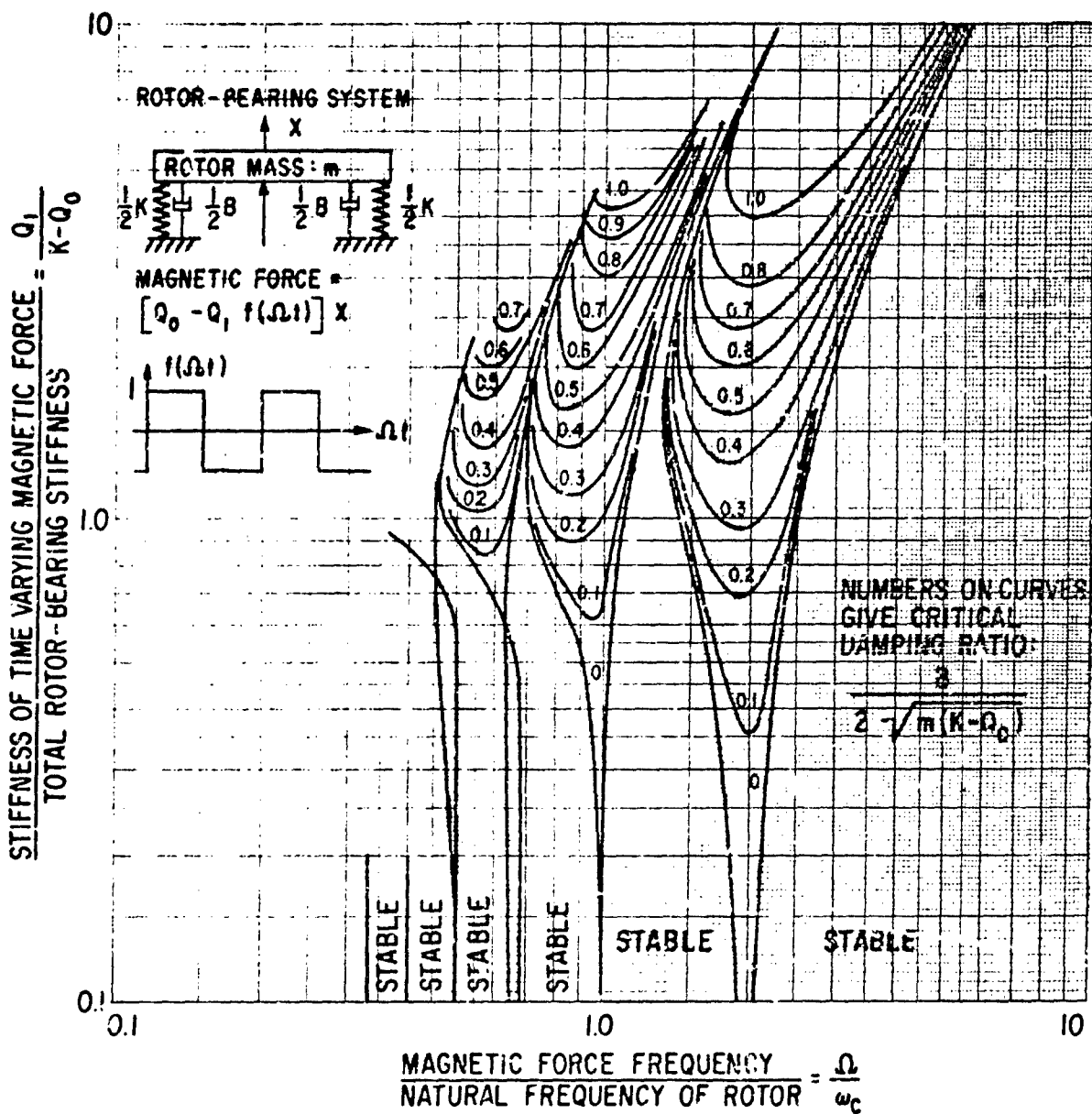
Figure 6   Stability Map For A Rigid Rotor With A "Square Wave"
Varying Magnetic Force

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APPENDIX I: Magnetic Forces of a Homopolar Generator Operating With No Load

A homopolar generator is shown schematically in Fig. 1. It is a brushless generator whose field coil is located between the plane of the north poles and the plane of the south poles. Let there be n north poles and n southpoles. Furthermore, the field coil has N_f windings such that it's mmf, $\mathfrak{I}_f$ is:

$$\mathfrak{I}_f = N_f\, i_f \tag{A.1}$$

where i_f is the current in the coil.

The magnetic reluctance of the airgap at the k'th north pole is $\mathcal{R}_{Nk}$ and at the k'th south pole $\mathcal{R}_{Sk}$. Set:

$$\frac{1}{\mathcal{R}_N} = \sum_{k=1}^{n} \frac{1}{\mathcal{R}_{Nk}} \tag{A.2}$$

$$\frac{1}{\mathcal{R}_S} = \sum_{k=1}^{n} \frac{1}{\mathcal{R}_{Sk}} \tag{A.3}$$

The flux, φ leaving the rotor through the north poles is the same flux that enters the rotor through the south poles. When the mmf's across the airgaps of the north poles and the south poles are $\mathfrak{I}_N$ and $\mathfrak{I}_S$, respectively, the equations relating flux and mmf becomes:

$$\varphi = \frac{\mathfrak{I}_N}{\mathcal{R}_N} = \frac{\mathfrak{I}_S}{\mathcal{R}_S} \tag{A.4}$$

Since $\mathcal{R}_N$ and $\mathcal{R}_S$ are in series, the total reluctance of the flux path is $(\mathcal{R}_N + \mathcal{R}_S)$, ignoring the reluctance of the iron (i.e. saturation effects are ignored). Hence:

$$\varphi = \frac{\mathfrak{I}_f}{\mathcal{R}_N + \mathcal{R}_S} \tag{A.5}$$

Combining eqs. (A.4) and (A.5):

$$\mathfrak{I}_N = \frac{\mathcal{R}_N}{\mathcal{R}_N + \mathcal{R}_S}\, \mathfrak{I}_f \tag{A.6}$$

$$\mathfrak{I}_S = \frac{\mathcal{R}_S}{\mathcal{R}_N + \mathcal{R}_S}\, \mathfrak{I}_f \tag{A.7}$$

To determine the reluctances, assume the rotor to be eccentric by the distance e from the center of the stator and let the angle between the direction of displacement and the vertical axis (the x-axis) be α:

center of stator — | ————→ y

ϵ

α ⟍ center of rotor

ω

Angular speed of rotor

x
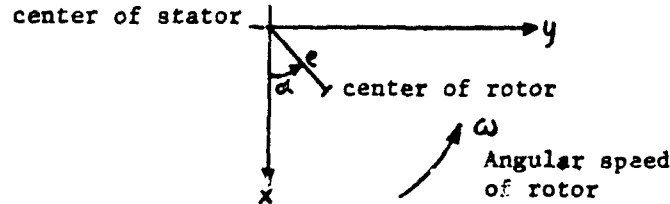
Define the eccentricity ratio ε by:

$$\varepsilon = \frac{e}{C} \tag{A.8}$$

where C is the mean radial gap at the poles. Hence:

$$x = e\cos\alpha = C\varepsilon\cos\alpha \tag{A.9}$$

$$y = e\sin\alpha = C\varepsilon\sin\alpha \tag{A.10}$$

At time t=0, the first northpole is on the x-axis. Hence the center of the k'th pole is at an angle $(k-1)\frac{2\pi}{n}$ from the x-axis at t=0. When the angular speed of the rotor is ω, the airgap at the center of the k'th northpole can be expressed as:

$$h_{Nk} = C\left[1 - \varepsilon\cos\left(\omega t - \alpha + \frac{2\pi}{n}(k-1)\right)\right] \tag{A.11}$$

The southpoles are displaced $\frac{\pi}{n}$ from the northpoles, and the airgap at the center of the k'th southpole becomes:

$$h_{Sk} = C\left[1 - \varepsilon\cos\left(\omega t - \alpha + \frac{\pi}{n} + \frac{2\pi}{n}(k-1)\right)\right] \tag{A.12}$$

Thus, the reluctance of the airgap of the k'th northpole becomes:

$$R_{Nk} = \frac{h_{Nk}}{A\mu} = \frac{C}{A\mu}\left[1 - \varepsilon\cos\left(\omega t - \alpha + \frac{2\pi}{n}(k-1)\right)\right] \tag{A.13}$$

where A is the area of a pole and μ is the permeability. If ε is assumed small ($\varepsilon << 1$), eq. (A.13) yields:

$$\frac{1}{R_{Nk}} = \frac{A\mu}{C} \left[1 + \varepsilon \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1)) \right]$$ (A.14)

Hence, from eq. (A.2):

$$\frac{1}{R_N} = \frac{A\mu}{C} \left[n + \varepsilon \sum_{k=1}^{n} \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1)) \right]$$

$$= \frac{A\mu}{C} \left[n + \varepsilon \cos(\omega t - \alpha) \sum_{k=1}^{n} \cos(\frac{2\pi}{n}(k-1)) - \varepsilon \sin(\omega t - \alpha) \sum_{k=1}^{n} \sin(\frac{2\pi}{n}(k-1)) \right]$$

Now:

$$\sum_{k=1}^{n} \cos(k \frac{2\pi}{n}) = \begin{cases} 1 & \text{for} \quad n=1 \\ 0 & \text{for} \quad n \geq 2 \end{cases}$$ (A.16)

$$\sum_{k=1}^{n} \sin(k \frac{2\pi}{n}) = 0$$ (A.17)

Ignoring the case of n=1, where there are no magnetic forces anyway, eq. (A.15) becomes:

$$R_N = \frac{C}{nA\mu}$$ (A.18)

Similarly, it is found that:

$$R_S = \frac{C}{nA\mu}$$ (A.19)

whereby eqs. (A.6) and (A.7) yield:

$$\mathfrak{Z}_N = \mathfrak{Z}_S = \frac{1}{2} \mathfrak{Z}_f$$ (A.20)

Since both $\mathfrak{Z}_N$ and $\mathfrak{Z}_S$, and also R_N and R_S are independent of time, the total flux ϕ will also be independent of time (see eq. (A.4)) which means that there is no self-induced current in the field coil.

On this basis the flux for the k'th northpole becomes:

$$\varphi_{Nk} = \frac{\Im_N}{\mathcal{R}_{Nk}} = \frac{\Im_k A \mu}{2C}\left[1 + \varepsilon \cos\left(\omega t - \alpha + \frac{2\pi}{n}(k-1)\right)\right]$$ (A.21)

and for the k'th southpole:

$$\varphi_{Sk} = \frac{\Im_S}{\mathcal{R}_{Sk}} = \frac{\Im_k A \mu}{2C}\left[1 + \varepsilon \cos\left(\omega t - \alpha + \frac{\pi}{n} + \frac{2\pi}{n}(k-1)\right)\right]$$ (A.22)

At each pole there is a radial force pulling on the rotor. For the k'th northpole this force becomes:

$$\bar{Q}\left(\frac{\varphi_{Nk}}{A}\right)^2 A$$

where $\bar{Q}$ is a constant which depends on the units employed for the quantities. If φ is in lines, A is in inch^2 and the force is measured in lbs., then

$$\bar{Q} = \frac{1}{72,130,000} \quad .$$

This force has x and y components which for the k'th northpole become:

$$\left(F_{Nx}\right)_k = \bar{Q}\,\frac{\varphi_{Nk}^2}{A}\cos\left(\omega t + \frac{2\pi}{n}(k-1)\right)$$ (A.23)

$$\left(F_{Ny}\right)_k = \bar{Q}\,\frac{\varphi_{Nk}^2}{A}\sin\left(\omega t + \frac{2\pi}{n}(k-1)\right)$$ (A.24)

When φ_{Nk} is substituted from eq. (A.21) and the forces are summed over all n northpoles, the total forces acting on the rotor in the plane of the northpoles become:

$$F_{Nx} = \sum_{k=1}^{n}\left(F_{Nx}\right)_k = \bar{Q}\,\frac{\Im_k^2 A \mu^2}{4C^2}\sum_{k=1}^{n}\left[1 + 2\varepsilon \cos\left(\omega t - \alpha + \frac{2\pi}{n}(k-1)\right)\right]\cos\left(\omega t + \frac{2\pi}{n}(k-1)\right)$$ (A.25)

$$F_{Ny} = \sum_{k=1}^{n}\left(F_{Ny}\right)_k = \bar{Q}\,\frac{\Im_k^2 A \mu^2}{4C^2}\sum_{k=1}^{n}\left[1 + 2\varepsilon \cos\left(\omega t - \alpha + \frac{2\pi}{n}(k-1)\right)\right]\sin\left(\omega t + \frac{2\pi}{n}(k-1)\right)$$ (A.26)

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By expanding the trigonometric functions and making use of eqs. (A.16) and (A.17), these equations can be written:

$$F_{Nx} = \bar{Q} \frac{3^2 A \mu}{4 C^2} \varepsilon \sum_{k=1}^{n} \left\{ \cos\alpha \left[1 + \cos 2 \left(\omega t + \frac{2\pi}{n}(k-1) \right) \right] + \sin\alpha \cdot \sin 2 \left(\omega t + \frac{2\pi}{n}(k-1) \right) \right\} \qquad (A.27)$$

$$F_{Ny} = \bar{Q} \frac{3^2 A \mu}{4 C^2} \varepsilon \sum_{k=1}^{n} \left\{ \cos\alpha \cdot \sin 2 \left(\omega t + \frac{2\pi}{n}(k-1) \right) + \sin\alpha \left[1 - \cos 2 \left(\omega t + \frac{2\pi}{n}(k-1) \right) \right] \right\} \qquad (A.28)$$

The following identities hold true:

$$\sum_{k=1}^{n} \cos\left(k \frac{4\pi}{n} \right) = \begin{cases} 1 & \text{for} \quad n=1 \\ 2 & \text{for} \quad n=2 \\ 0 & \text{for} \quad n \geq 3 \end{cases} \qquad (A.29)$$

$$\sum_{k=1}^{n} \sin\left(k \frac{4\pi}{n} \right) = 0 \qquad \text{for all} \quad n \qquad (A.30)$$

Furthermore, since the total flux is φ the average flux density B_0 is:

$$B_0 = \frac{\varphi}{nA} = \frac{3_f}{nA(\mathcal{R}_N + \mathcal{R}_s)} = \frac{\mu 3_f}{2C} \qquad (A.31)$$

Introducing these equations into eqs. (A.27) and (A.28) and making use of eqs. (A.9) and (A.10), the result becomes:

For n=2

$$F_{Nx} = 2\bar{Q} \frac{A B_0^2}{C} \left[x_N (1 + \cos(2\omega t)) + y_N \sin(2\omega t) \right] \qquad (A.32)$$

$$F_{Ny} = 2\bar{Q} \frac{A B_0^2}{C} \left[x_N \sin(2\omega t) + y_N (1 - \cos(2\omega t)) \right] \qquad (A.33)$$

For n≥3

$$F_{Nx} = n\bar{Q} \frac{A B_0^2}{C} x_N \qquad (A.34)$$

$$F_{Ny} = n\bar{Q} \frac{A B_0^2}{C} y_N \qquad (A.35)$$

Similarly, the forces in the plane of the southpoles become:

For n=2

$$F_{Sx} = 2\bar{Q} \frac{A B_0^2}{C} \left[x_S (1 - \cos(2\omega t)) - y_S \sin(2\omega t) \right]$$ (A.36)

$$F_{Sy} = 2\bar{Q} \frac{A B_0^2}{C} \left[-x_S \sin(2\omega t) + y_S (1 + \cos(2\omega t)) \right]$$ (A.37)

For n≥3

$$F_{Sx} = n\bar{Q} \frac{A B_0^2}{C} x_S$$ (A.38)

$$F_{Sy} = n\bar{Q} \frac{A B_0^2}{C} y_S$$ (A.39)

For use in the stability and response calculation, these forces should be written in a different form. Let the distance between the pole planes be L_p and let the rotor displacement in the center between the two planes be x and y. Furthermore, let the rotor have the slopes $\Theta = \frac{dx}{dz}$ and $\varphi = \frac{dy}{dz}$ where z is the axial coordinate. Then the displacements in the pole planes become:

$$x_N = x + \tfrac{1}{2} L_p \Theta \qquad\qquad y_N = y + \tfrac{1}{2} L_p \varphi$$

$$x_S = x - \tfrac{1}{2} L_p \Theta \qquad\qquad y_S = y - \tfrac{1}{2} L_p \varphi$$ (A.40)

The forces and moments acting on the rotor become:

$$F_x = F_{Nx} + F_{Sx} \qquad\qquad F_y = F_{Ny} + F_{Sy}$$

$$T_x = \tfrac{1}{2} L_p (F_{Nx} - F_{Sx}) \qquad\qquad T_y = \tfrac{1}{2} L_p (F_{Ny} - F_{Sy})$$ (A.41)

Substitute eqs. (A.40) into eqs. (A.32) to (A.39) and combine them according to eq. (A.41) to get:

For n=2

$$F_x = 2\bar{Q} \frac{A B_0^2}{C} \left[2x + \Theta L_p \cos(2\omega t) + \varphi L_p \sin(2\omega t) \right]$$ (A.42)

$$F_y = 2\bar{Q} \frac{A B_0^2}{C} \left[2y + \Theta L_p \sin(2\omega t) - \varphi L_p \cos(2\omega t) \right]$$ (A.43)

$$T_x = 2\bar{Q} \frac{A B_0^2}{C} \left[x L_p \cos(2\omega t) + y L_p \sin(2\omega t) + \tfrac{1}{2} L_p^2 \Theta \right]$$ (A.44)

$$T_y = 2\bar{Q} \frac{A B_0^2}{C} \left[x L_p \sin(2\omega t) - y L_p \cos(2\omega t) + \tfrac{1}{2} L_p^2 \varphi \right]$$ (A.45)

which can be written in matrix form:

For n=2

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \begin{Bmatrix} Q_0 x \\ Q_0 y \\ Q_0' \theta \\ Q_0' \varphi \end{Bmatrix} - \left[Q \cos(2\omega t) - q \sin(2\omega t) \right] \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix} \qquad (A.46)$$

Here:

For n=2

$$Q_0 = 4\bar{Q}\,\frac{AB_0^2}{C} \qquad \frac{lbs}{inch} \qquad (A.47)$$

$$Q_0' = \bar{Q}\,\frac{AB_0^2}{C}\,L_P^2 \qquad \frac{lbs \cdot inch}{radian} \qquad (A.48)$$

and Q and q are 4 by 4 matrices:

For n=2

$$Q = 2\bar{Q}\,\frac{AB_0^2}{C}\,L_P \begin{Bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{Bmatrix} \qquad lbs \qquad (A.49)$$

$$q = 2\bar{Q}\,\frac{AB_0^2}{C}\,L_P \begin{Bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{Bmatrix} \qquad lbs \qquad (A.50)$$

In this form the results can be used directly in the stability and response calculations. When there are more than four poles (i.e. $n \geq 3$) the forces are not time dependent and, hence, a stability or response calculation of the type under investigation does not apply. However, there will still be negative lateral and moment stiffnesses which must be taken into account when performing the more conventional rotor unbalance response calculation.

These spring coefficients are:

For n≥3

$$Q_0 = 2n\bar{Q}\,\frac{AB_0^2}{C} \qquad \frac{lbs}{inch}$$

$$Q_0' = \tfrac{1}{2}n\bar{Q}\,\frac{AB_0^2}{C}\,L_P^2 \qquad \frac{lbs \cdot inch}{radian}$$

Magnetic Forces of a Heteropolar Inductor Generator Operating with No Load

A cross-section of a heteropolar generator is shown schematically in Fig. 2. It is a brushless generator with a field coil for each pole such that a pole receives its flux from two field coils. Each pole has two faces which are provided with teeth. The rotor likewise has teeth. Schematically, with 2 north poles and 2 south poles the magnetic circuit can be shown as:
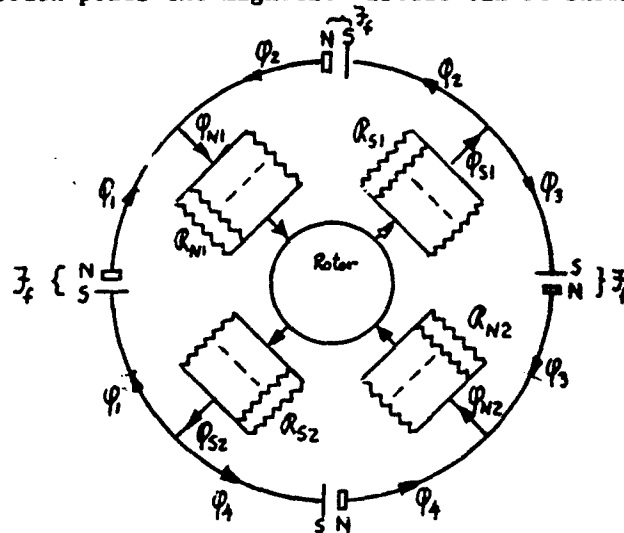
Figure 7: Schematic Diagram Showing the Magnetic
Circuit of a Heteropolar Generator

Here, φ_1 is the flux generated by field coil No. 1, φ_2 is the flux generated by field coil No. 2, and so on. φ_1 and φ_2 combine and make up the flux, φ_{N1} passing through the first north pole, and this flux returns from the rotor to the stator through the south poles as part of φ_{S1} and φ_{S2} .

Similarly, φ_3 and φ_4 combine to the flux φ_{N2} passing through the second north pole, and so on.

The reluctances of the airgaps at the poles are $\mathcal{R}_{N1}$, $\mathcal{R}_{S1}$, $\mathcal{R}_{N2}$ and $\mathcal{R}_{S2}$. The mmf's across these airgaps are $\mathcal{F}_{N1}$, $\mathcal{F}_{S1}$, $\mathcal{F}_{N2}$ and $\mathcal{F}_{S2}$, respectively.

Since the sum of all mmf's around any closed circuit has to equal zero, Fig. 7 shows that:

$$\mathfrak{F}_{N1} + \mathfrak{F}_{S1} = \mathfrak{F}_{S1} + \mathfrak{F}_{N2} = \mathfrak{F}_{N2} + \mathfrak{F}_{S2} = \mathfrak{F}_{S2} + \mathfrak{F}_{N1} = \mathfrak{F}_f \tag{B.1}$$

ffom which:

$$\left. \begin{array}{c} \mathfrak{F}_{N1} = \mathfrak{F}_{N2} = \mathfrak{F}_N \\ \mathfrak{F}_{S1} = \mathfrak{F}_{S2} = \mathfrak{F}_S \end{array} \right\} \quad \mathfrak{F}_N + \mathfrak{F}_S = \mathfrak{F}_f \tag{B.2}$$

In general, the generator has n north poles and n south poles, in which case:

$$\left. \begin{array}{c} \mathfrak{F}_{N1} = \mathfrak{F}_{N2} = - - - = \mathfrak{F}_{Nn} = \mathfrak{F}_N \\ \mathfrak{F}_{S1} = \mathfrak{F}_{S2} = - - - = \mathfrak{F}_{Sn} = \mathfrak{F}_S \end{array} \right\} \quad \mathfrak{F}_N + \mathfrak{F}_S = \mathfrak{F}_f \tag{B.3}$$

The flux across the pole airgaps then becomes:

$$\varphi_{N1} = \varphi_1 + \varphi_2 = \frac{\mathfrak{F}_{N1}}{R_{N1}}$$

$$\varphi_{S1} = \varphi_2 + \varphi_3 = \frac{\mathfrak{F}_{S1}}{R_{S1}}$$

$$\varphi_{Nn} = \varphi_{2n-1} + \varphi_{2n} = \frac{\mathfrak{F}_{Nn}}{R_{Nn}}$$

$$\varphi_{Sn} = \varphi_{2n} + \varphi_1 = \frac{\mathfrak{F}_{Sn}}{R_{Sn}} \tag{B.4}$$

The total flux, φ, is given by:

$$\varphi = \sum_{k=1}^{2n} \varphi_k$$

Thus, from eq. (B.4):

$$\varphi = \left(\frac{1}{R_{N1}} + \frac{1}{R_{N2}} + - - - + \frac{1}{R_{Nn}} \right) \mathfrak{F}_N = \left(\frac{1}{R_{S1}} + \frac{1}{R_{S2}} + - - + \frac{1}{R_{Sn}} \right) \mathfrak{F}_S \tag{B.5}$$

Set:

$$\frac{1}{R_N} = \sum_{k=1}^{n} \frac{1}{R_{Nk}}$$

$$\frac{1}{R_S} = \sum_{k=1}^{n} \frac{1}{R_{Sk}} \tag{B.6}$$

Then:

$$\mathfrak{F}_N = R_N \varphi$$

$$\mathfrak{F}_S = R_S \varphi \tag{B.7}$$

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which by means of eq. (B.3) yields:

$$\mathfrak{Z}_f = \mathfrak{Z}_N + \mathfrak{Z}_S = (\mathcal{R}_N + \mathcal{R}_S)\,\varphi \tag{B.8}$$

or:

$$\mathfrak{Z}_N = \frac{\mathcal{R}_N}{\mathcal{R}_N + \mathcal{R}_S}\,\mathfrak{Z}_f$$

$$\mathfrak{Z}_S = \frac{\mathcal{R}_S}{\mathcal{R}_N + \mathcal{R}_S}\,\mathfrak{Z}_f \tag{B.9}$$

To determine the reluctances, consider the airgaps at a pole:
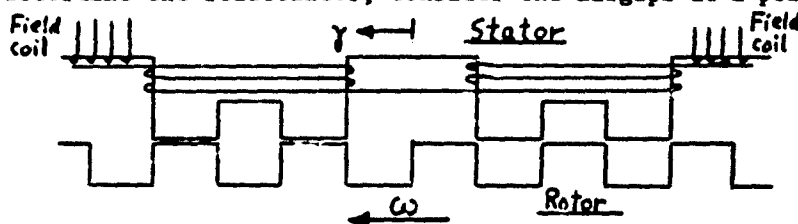
Figure 8: Pole Airgaps At Time t=0

Let there be n_r rotor teeth in total. Then a rotor tooth or a stator tooth extends over an angle $\frac{\pi}{n_r}$. Thus, if the angle γ measured from the center-line of the pole, is fixed in the stator, the position of the centers of the stator teeth are:

$$\gamma_j = \pm 3\frac{\pi}{2n_r}, \pm 7\frac{\pi}{2n_r}, \;-\;-\;-\;-\;, \pm(4j-1)\frac{\pi}{2n_r} \tag{B.10}$$

when there are $2n_s$ stator teeth per pole. Note:

$$n_r = 2n\,(2n_s + 1) \tag{B.11}$$

Introduce an x-axis which passes between the last south pole (number n) and the first north pole. Measured from this axis the centerline of the k'th pole is located at:

$$\frac{\pi}{2n} + \frac{\pi}{n}(k-1) \qquad\qquad k = 1,2, \;-\;-\;-\;, 2n \tag{B.12}$$

Thus, the j'th stator tooth at the k'th pole is located an angle: $\frac{\pi}{2n} + \frac{\pi}{n}(k-1) + \gamma_j$

from the x-axis. Assume the rotor to be eccentric such that the center of the rotor is a distance $C\varepsilon$ from the stator center, and the angle α

is the angle between the x-axis and the direction of eccentricity. Hence:

$$x = C\varepsilon \cos\alpha$$

$$y = C\varepsilon \sin\alpha$$

(B.13)

where C is the radial clearance for the concentric rotor and ε is the eccentricity ratio. Thus, the airgap h_{kj} at the j'th stator tooth for the k'th pole becomes:

$$h_{kj} = C\left[1 - \varepsilon \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha\right)\right]$$

(B.14)

Next, let figure 8 apply to the time t=0. When the angular speed of the rotor is ω, the flux area of a stator tooth becomes:

$$A_j = \tfrac{1}{2} A_T \left(1 \pm \cos(n_r \omega t)\right) = \tfrac{1}{2} A_T \left(1 \pm \cos(\nu t)\right)$$

(B.15)

$$\text{Electrical frequency}: \nu = n_r \omega$$

(B.16)

where the plus sign applies to the teeth where γ_j is positive, and the minus sign where γ_j is negative. A_T is the actual area of a stator tooth.

From eqs. (B.14) and (B.15) the reluctance $\mathcal{R}_{kj}$ of the airgap at the j'th stator tooth of the k'th pole can be expressed as:

$$\frac{1}{\mathcal{R}_{kj}} = \frac{\mu A_j}{h_{kj}} = \frac{\mu A_T}{2C}\left(1 \pm \cos(\nu t)\right)\left[1 + \varepsilon \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha\right)\right]$$

(B.17)

since $\varepsilon \ll 1$. Let the total reluctance for positive γ_j be $\mathcal{R}_{ak}$ and for negative γ_j be $\mathcal{R}_{bk}$. Then:

$$\frac{1}{\mathcal{R}_{ak}} = \sum_{j=1}^{n_s}\left(\frac{1}{\mathcal{R}_{kj}}\right)_{\gamma_j > 0} = \frac{\mu A_T}{2C}\sum_{j=1}^{n_s}\left(1 + \cos(\nu t)\right)\left[1 + \varepsilon \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + (4j-1)\frac{\pi}{2n_r} - \alpha\right)\right]$$

$$= \frac{\mu A_T}{2C}\left(1 + \cos(\nu t)\right)\left[n_s + \varepsilon\left(G\cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right) - H\sin\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right)\right)\right]$$

(B.18)

$$\frac{1}{\mathcal{R}_{bk}} = \sum_{j=1}^{n_s}\left(\frac{1}{\mathcal{R}_{kj}}\right)_{\gamma_j < 0} = \frac{\mu A_T}{2C}\sum_{j=1}^{n_s}\left(1 - \cos(\nu t)\right)\left[1 + \varepsilon \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - (4j-1)\frac{\pi}{2n_r} - \alpha\right)\right]$$

$$\frac{\mu A_T}{2C}\left(1 - \cos(\nu t)\right)\left[n_s + \varepsilon\left(G\cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right) + H\sin\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right)\right)\right]$$

(B.19)

where:

$$G = \sum_{j=1}^{n_g} \cos(4j-1)\frac{\pi}{2n_r} \qquad (B.20)$$

$$H = \sum_{j=1}^{n_g} \sin(4j-1)\frac{\pi}{2n_r} \qquad (B.21)$$

The total reluctance, $\mathcal{R}_k$, of the airgaps at the k'th pole is then:

$$\frac{1}{\mathcal{R}_k} = \frac{1}{\mathcal{R}_{ak}} + \frac{1}{\mathcal{R}_{bk}} = \frac{\mu A_T}{C}\left[n_s + \varepsilon G \cos\left(\frac{\pi}{n}(k-1)+\frac{\pi}{2n}-\alpha\right) - \varepsilon H \cos(\nu t)\sin\left(\frac{\pi}{n}(k-1)+\frac{\pi}{2n}-\alpha\right)\right] \qquad (B.22)$$

The first north pole is at $k=1$, the second at $k=3$, and the last at $k=2n-1$. Thus, substitution of eq. (B.22) into eq. (B.6) yields:

$$\frac{1}{\mathcal{R}_N} = \sum_{k=1,3,\cdots(2n-1)}\frac{1}{\mathcal{R}_k} = \frac{\mu A_T}{C}nn_s \qquad (B.23)$$

where the following relationships have been employed:

$$\sum_{k=1,3,\cdots(2n-1)}\cos\left(\frac{\pi}{n}(k-1)\right) = \sum_{k=1}^{n}\cos\left(k\frac{2\pi}{n}\right) = \begin{cases} 1 & \text{for } n=1 \\ 0 & \text{for } n\geq2 \end{cases} \qquad (B.24)$$

$$\sum_{k=1,3,\cdots(2n-1)}\sin\left(\frac{\pi}{n}(k-1)\right) = \sum_{k=1}^{n}\sin\left(k\frac{2\pi}{n}\right) = 0 \qquad (B.25)$$

and the case of n=1 has been ignored as being of no interest. The first south pole is at k=2, the second at k=4 and the last at k=2n. Then, from eq. (B.22) and (B.6):

$$\frac{1}{\mathcal{R}_S} = \sum_{k=2,4,\cdots2n}\frac{1}{\mathcal{R}_k} = \frac{\mu A_T}{C}nn_s \qquad (B.26)$$

Therefore, $\mathcal{R}_S = \mathcal{R}_N$ and eq. (B.9) yields:

$$\mathcal{F}_N = \mathcal{F}_S = \frac{1}{2}\mathcal{F}_f \qquad (B.27)$$

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Having established the mmf across the pole airgaps, the flux density B_{kj} at the j'th stator tooth of the k'th pole becomes:

$$B_{kj} = \frac{\frac{1}{2}\mathcal{F}_t}{A_j \mathcal{R}_{kj}} = \frac{\mu \mathcal{F}_t}{2 h_{kj}} = B_o \left[1 + \varepsilon \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha\right) \right] \tag{B.28}$$

where:

$$B_o = \frac{\mu \mathcal{F}_t}{2C} \tag{B.29}$$

B_o is the average flux density. The force acting on the stator tooth has an x-component and a y-component which are determined by:

$$\left. \begin{matrix} (F_x)_{kj} \\ \\ (F_y)_{kj} \end{matrix} \right\} = \bar{Q} A_j B_{kj}^2 \cdot \begin{cases} \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) \\ \\ \sin\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) \end{cases} \tag{B.30}$$

where, from eq. (B.28):

$$B_{kj}^2 = B_o^2 \left[1 + 2\varepsilon \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha\right) \right]$$
$$= \frac{1}{C} B_o^2 \left[C + 2x \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) + 2y \sin\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) \right] \tag{B.31}$$

x and y are the rotor displacements from eq. (B.13). Substituting eq. (B.31) into eq. (B.30) there will appear the following products:

$$\cos^2\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) = \frac{1}{2}\left[1 + \cos 2\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) \right]$$
$$\sin^2\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) = \frac{1}{2}\left[1 - \cos 2\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) \right] \tag{B.32}$$
$$\cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) \sin\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right) = \frac{1}{2}\sin 2\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j\right)$$

Now, the total magnetic force components are given by:

$$F_x = \sum_{k=1}^{2n} \sum_{j=1}^{n_j} (F_x)_{kj} \qquad\qquad F_y = \sum_{k=1}^{2n} \sum_{j=1}^{n_j} (F_y)_{kj} \tag{B.33}$$

The following relationships hold true:

$$\sum_{k=1}^{2n} \cos\left(\frac{2\pi}{n}(k-1)\right) = \begin{cases} 2 & \text{for} \quad n=1 \\ 0 & \text{for} \quad n \geq 2 \end{cases} \tag{B.34}$$

$$\sum_{k=1}^{2n} \sin\left(\frac{2\pi}{n}(k-1)\right) = 0 \tag{B.35}$$

If these relationships are used together with the similar ones of eqs. (B.24) and (B.25), substitution of eq. (B.31) into (B.30) and summing according to eq. (B.33) yields:

$$F_x = \bar{Q} \frac{A_T B_o^2}{C} \frac{1}{2} \left[1 + \cos(\nu t) + 1 - \cos(\nu t) \right] 2 n n_s x = \bar{Q} \frac{2 n n_s A_T B_o^2}{C} x \tag{B.36}$$

$$F_y = \bar{Q} \frac{2 n n_s A_T B_o^2}{C} y \tag{B.37}$$

It is seen that the forces are purely static and are not dependent on time. Thus, the magnetic forces for a heteropolar generator with no load do not cause the rotor to whirl. However, they do contribute a negative stiffness:

$$Q_o = \bar{Q} \frac{2 n n_s A_T B_o^2}{C} \tag{B.38}$$

which must be taken into account if the unbalance response of the rotor is calculated or the hydrodynamic whirl instability is being checked. Of course, if Q_o exceeds the combined bearing stiffness, the rotor is statically unstable.

APPENDIX III:Magnetic Forces of a Two-Coil Lundell Generator

A two-coil Lundell generator is shown schematically in Figure 3. There are two
field coils in this generator, one on each side of the central plane which contains
the northpoles. The magnetic flux path goes from the northpoles of the rotor
(Fig. 3a) through the stator to the southpoles of the rotor (see Fig. 9), and
then through the cylindrical air gaps g_1 and g_3 (Fig. 3b), the stationary pieces
on which the field coils are wound, the conical airgaps g_2 and g_4 and back to
the northpoles of the rotor. The magnetic circuit is shown diagrammatically
in Fig. 10. The reluctances of the airgaps g_1, g_2, g_3 and g_4 are respectively
represented by $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$ and $\mathcal{R}_4$. Each field winding produces an m.m.f.
of $\mathcal{F}_f$. Let there be n north poles and n south poles, and let $\mathcal{R}_N$ and $\mathcal{R}_S$ be the
total reluctances of the airgaps of the north poles and the south poles, re-
spectively. From the magnetic circuit shown in Fig. 10 we have:

$$\mathcal{F}_f = \varphi(\mathcal{R}_N + \mathcal{R}_S) + \varphi_1(\mathcal{R}_1 + \mathcal{R}_2) \tag{C.1}$$

$$\varphi_1(\mathcal{R}_1 + \mathcal{R}_2) = \varphi_3(\mathcal{R}_3 + \mathcal{R}_4) \tag{C.2}$$

$$\varphi = \varphi_1 + \varphi_3 \tag{C.3}$$
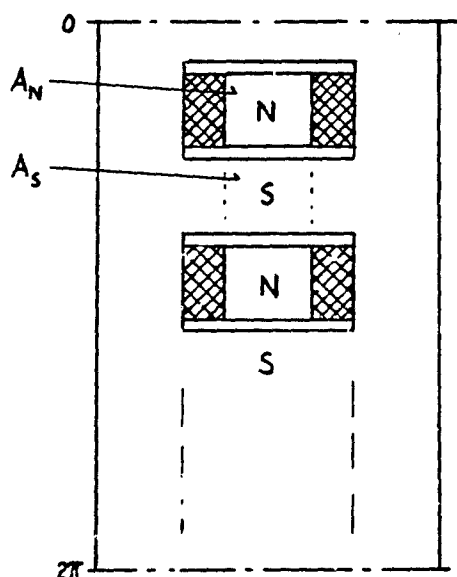
Figure 9

Expanded view of outer
surface of rotor.
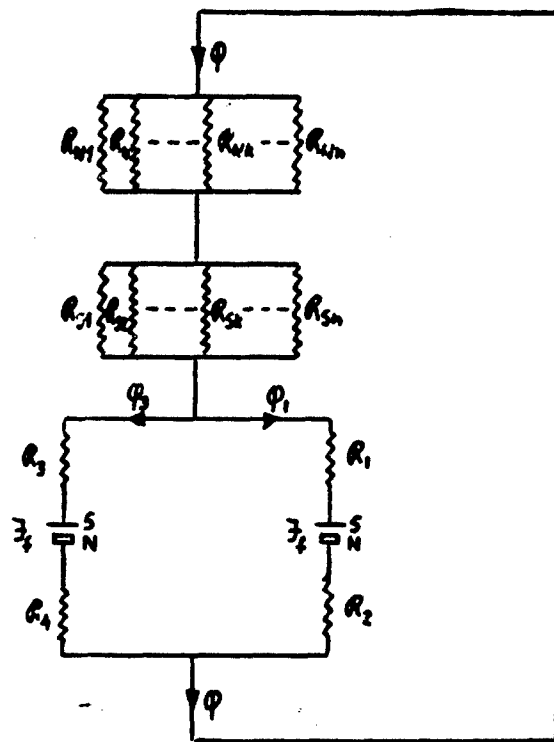
Figure 10

Magnetic Circuit for a Two-Coil Lundell Generator
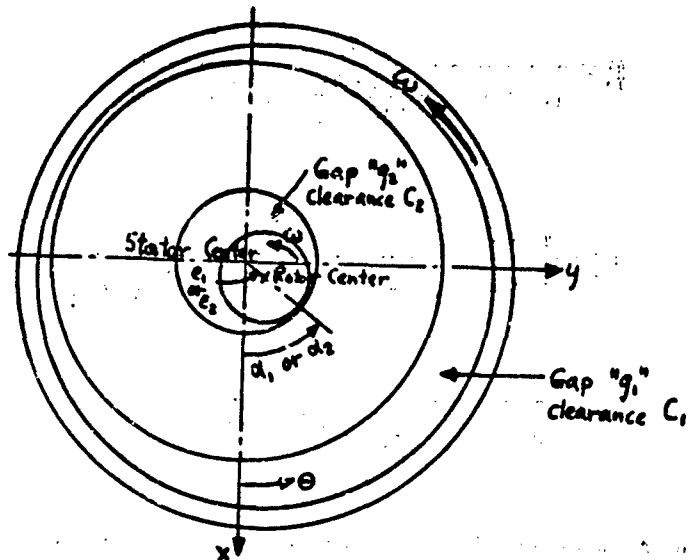
Figure 11

Section "R-R" of Fig. 3b
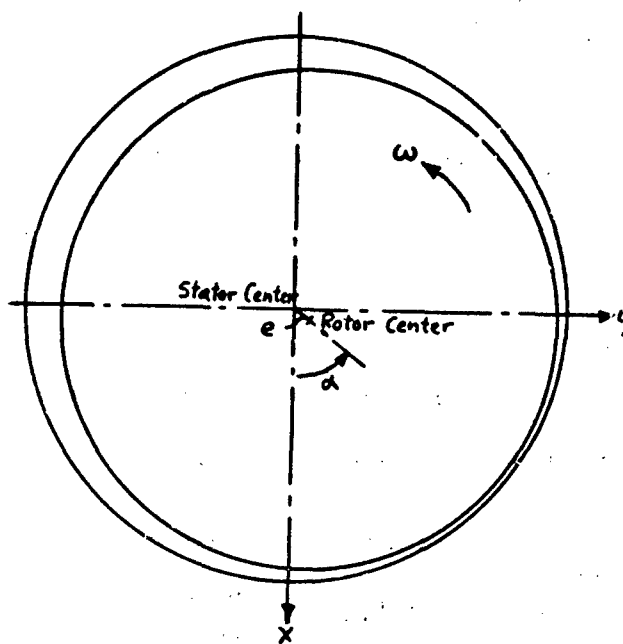
Figure 12

Section "C-C" of Fig. 3b

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Solving the above three equations for φ_1, φ_3 and φ, we obtain:

$$\varphi_1 = \frac{\mathcal{F}_f}{(R_1 + R_2) + \left(\frac{R_1 + R_2}{R_3 + R_4} + 1\right)(R_N + R_S)} \tag{C.4}$$

$$\varphi_3 = \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{\mathcal{F}_f}{(R_1 + R_2) + \left(\frac{R_1 + R_2}{R_3 + R_4} + 1\right)(R_N + R_S)} \tag{C.5}$$

$$\varphi = \frac{\mathcal{F}_f}{(R_1 + R_2)\left(\frac{R_1 + R_2}{R_3 + R_4} + 1\right)^{-1} + (R_N + R_S)} \tag{C.6}$$

To calculate the reluctance, R_1, of the airgap g_1, let us consider the airgap as infinitely many reluctances connected in parallel. Thus, referring to Figure 11, we have:

$$\frac{1}{R_1} = \int_0^{2\pi} \frac{\mu L_1 r \, d\theta}{h_1} \tag{C.7}$$

where: h_1=film thickness of airgap $g_1 = C_1 \left[1 + \varepsilon_1 \cos(\theta - d_1)\right]$

$$\varepsilon_1 = \frac{e_1}{C_1} \tag{C.8}$$

L_1=axial length of airgap g_1

e_1 and C_1 are the eccentricity and the mean film thickness of airgap g_1,

r is the mean radius of the cylindrical airgap

Substituting eq. (C.8) into (C.7) and neglecting terms of the order ε_1^2:

$$\frac{1}{R_1} = \frac{\mu L_1 r}{C_1} \int_0^{2\pi} \left[1 - \varepsilon_1 \cos(\theta - d_1)\right] d\theta = \frac{2\pi \mu L_1 r}{C_1}$$

or:

$$R_1 = \frac{C_1}{\mu L_1 2\pi r} \tag{C.9}$$

Similarly, the airgap g_3 has the reluctance:

$$R_3 = \frac{C_3}{\mu L_1 \, 2\pi r} \tag{C.10}$$

and for the conical airgaps g_2 and g_4,

$$R_2 = \frac{C_2}{\mu L_2 \, 2\pi \bar{r}} \tag{C.11}$$

$$R_4 = \frac{C_4}{\mu L_2 \, 2\pi \bar{r}} \tag{C.12}$$

where $\bar{r}$ = arithmetic mean radius. In general, because of manufacturing tolerances:

$$C_1 \neq C_3$$

$$C_2 \neq C_4 \tag{C.13}$$

Let R_{Nk} be the reluctance of the airgap at the k'th north pole:

$$R_{Nk} = \frac{C}{\mu A_N}\left[1 - \varepsilon \cos(\Theta_{Nk} + \omega t - \alpha)\right] \tag{C.14}$$

Similarly,

$$R_{Sk} = \frac{C}{\mu A_S}\left[1 - \varepsilon \cos(\Theta_{Sk} + \omega t - \alpha)\right] \tag{C.15}$$

where:

$$\Theta_{Nk} = \frac{2\pi}{n}(k-1)$$
$$\Theta_{Sk} = \frac{2\pi}{n}(k-1) + \frac{\pi}{n} \qquad\qquad \varepsilon = \frac{e}{C} \tag{C.16}$$

Now:

$$\frac{1}{R_N} = \sum_{k=1}^{n}\frac{1}{R_{Nk}} = \frac{\mu A_N}{C}\sum_{k=1}^{n}\left[1 + \varepsilon\cos(\Theta_{Nk} + \omega t - \alpha)\right] = \begin{cases} \dfrac{\mu A_N}{C}\left[1 + \varepsilon\cos(\omega t - \alpha)\right] & \text{for } n=1 \\[2mm] n\dfrac{\mu A_N}{C} & \text{for } n \geq 2 \end{cases} \tag{C.17}$$

By the same procedure:

$$\frac{1}{R_S} = \begin{cases} \dfrac{\mu A_S}{C}\left[1 + \varepsilon\cos(\omega t + \pi - \alpha)\right] & \text{for } n=1 \\[2mm] n\dfrac{\mu A_S}{C} & \text{for } n \geq 2 \end{cases} \tag{C.18}$$

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From here on, we assume that the generator has at least two pairs of poles. Thus for $n \geq 2$:

$$\mathcal{R}_N = \frac{C}{\mu n A_N}$$

$$\mathcal{R}_s = \frac{C}{\mu n A_s} \tag{C.19}$$

So far, we have obtained $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$ and $\mathcal{R}_4$ (Eqs. (C.9) to (C.12), and $\mathcal{R}_N$ and $\mathcal{R}_s$ (Eq. (C.19)). Hence for a given field m.m.f. $\mathfrak{F}_f$ we can calculate φ_1, φ_3 and φ from Eqs. (C.4) to (C.6). The magnetic flux through the individual north and southpoles can be expressed by:

$$\varphi_{Nk} = \frac{\mathcal{R}_N}{\mathcal{R}_{Nk}} \varphi \tag{C.20}$$

$$\varphi_{Sk} = \frac{\mathcal{R}_s}{\mathcal{R}_{Sk}} \varphi \tag{C.21}$$

Using eqs. (C.14) and (C.15), and for small ε, we obtain:

$$\varphi_{Nk} = \frac{\varphi}{n} \left[1 + \varepsilon \cos\left(\Theta_{Nk} + \omega t - \alpha \right) \right] \qquad n \geq 2 \tag{C.22}$$

$$\varphi_{Sk} = \frac{\varphi}{n} \left[1 + \varepsilon \cos\left(\Theta_{Sk} + \omega t - \alpha \right) \right] \qquad n \geq 2 \tag{C.23}$$

The x and y components of the magnetic force are:

$$F_{Nx} = \frac{\bar{Q}}{A_N} \sum_{k=1}^{n} \varphi_{Nk}^2 \cos(\Theta_{Nk} + \omega t) = \frac{\bar{Q}}{A_N} \left(\frac{\varphi}{n} \right)^2 \sum_{k=1}^{n} \left[1 + 2\varepsilon \cos(\Theta_{Nk} + \omega t - \alpha) \right] \cos(\Theta_{Nk} + \omega t) \tag{C.24}$$

$$F_{Ny} = \frac{\bar{Q}}{A_s} \left(\frac{\varphi}{n} \right)^2 \sum_{k=1}^{n} \left[1 + 2\varepsilon \cos(\Theta_{Nk} + \omega t - \alpha) \right] \sin(\Theta_{Nk} + \omega t) \tag{C.25}$$

Using the relationships:

$$\sum_{k=1}^{n} \cos\left(\frac{2\pi}{n}(k-1)\right) = \begin{cases} 1 & \text{for } n=1 \\ 0 & \text{for } n \geq 2 \end{cases}$$

(C.26)

$$\sum_{k=1}^{n} \sin\left(\frac{2\pi}{n}(k-1)\right) = 0$$

$$\sum_{k=1}^{n} \cos\left(\frac{4\pi}{n}(k-1)\right) = \begin{cases} 1 & \text{for } n=1 \\ 2 & \text{for } n=2 \\ 0 & \text{for } n \geq 3 \end{cases}$$

$$\sum_{k=1}^{n} \sin\left(\frac{4\pi}{n}(k-1)\right) = 0$$

(C.27)

eq. (C.24) is readily reduced to

$$F_{Nx} = \frac{\bar{Q}}{A_N}\left(\frac{\varphi}{n}\right)^2 \sum_{k=1}^{n} 2\varepsilon \cos(\Theta_{Nk}+\omega t-\alpha)\cos(\Theta_{Nk}+\omega t)$$

$$= \frac{\bar{Q}}{A_N}\left(\frac{\varphi}{n}\right)^2 \varepsilon \sum_{k=1}^{n}\left\{\cos(2\Theta_{Nk}+2\omega t-\alpha)+\cos\alpha\right\}$$

$$= \frac{\bar{Q}}{A_N}\left(\frac{\varphi}{n}\right)^2 \varepsilon \sum_{k=1}^{n}\left\{\cos\alpha +(\cos\alpha\cdot\cos(2\omega t)+\sin\alpha\sin(2\omega t))\cos(2\Theta_{Nk})\right\}$$

Thus,

$$F_{Nx} = \begin{cases} \frac{\bar{Q}}{A_N}\left(\frac{\varphi}{2}\right)^2 2\varepsilon\left[\cos\alpha(1+\cos(2\omega t))+\sin\alpha\cdot\sin(2\omega t)\right] & \text{for } n=2 \\ \\ n\frac{\bar{Q}}{A_N}\left(\frac{\varphi}{n}\right)^2 \varepsilon\cos\alpha & \text{for } n \geq 3 \end{cases}$$

(C.28)

Similarly,

$$F_{Ny} = \begin{cases} \frac{\bar{Q}}{A_N}\left(\frac{\varphi}{2}\right)^2 2\varepsilon\left[\cos\alpha\cdot\sin(2\omega t)+\sin\alpha(1-\cos(2\omega t))\right] & \text{for } n=2 \\ n\frac{\bar{Q}}{A_n}\left(\frac{\varphi}{n}\right)^2 \varepsilon\sin\alpha & \text{for } n \geq 3 \end{cases}$$

(C.29)

$$F_{Sx} = \begin{cases} \frac{\bar{Q}}{A_s}\left(\frac{\varphi}{2}\right)^2 2\varepsilon\left[\cos\alpha(1-\cos(2\omega t))-\sin\alpha\cdot\sin(2\omega t)\right] & \text{for } n=2 \\ n\frac{\bar{Q}}{A_s}\left(\frac{\varphi}{n}\right)^2 \varepsilon\cos\alpha & \text{for } n \geq 3 \end{cases}$$

(C.30)

$$F_{Sy} = \begin{cases} \frac{\bar{Q}}{A_s}\left(\frac{\varphi}{2}\right)^2 2\varepsilon\left[-\cos\alpha\sin(2\omega t)+\sin\alpha(1+\cos(2\omega t)\right] & \text{for } n=2 \\ n\frac{\bar{Q}}{A_s}\left(\frac{\varphi}{n}\right)^2 \varepsilon\sin\alpha & \text{for } n \geq 3 \end{cases}$$

(C.31)

Thus, it is seen that for generators with at least three pairs of poles, the magnetic forces due to the north and south poles are time-independent. For n = 2, the magnetic forces are functions of time as indicated by the above equations, if $A_N \neq A_S$. If, however, $A_N = A_S$ and (n=2), then the time-dependent parts of the north and southpoles cancel with each other, and the resultant ($F_{Nx} + F_{Sx}$) and ($F_{Ny} + F_{Sy}$) are again time-independent.

If the displacements of the rotor center are x and y, it is seen from Fig. 12 and so on that:

$$x = e \cos\alpha = C_E \cos\alpha$$

$$y = e \sin\alpha = C_E \sin\alpha$$

Furthermore, introduce the flux densities:

$$B_N = \frac{\varphi}{h A_N}$$

$$B_S = \frac{\varphi}{h A_S}$$

Then eqs. (C.28) to (C.31) can be written:

$$F_{Nx} = \begin{cases} 2\bar{Q}\,\frac{A_N B_N^2}{C}\,\big[x(1+\cos(2\omega t)) + y\sin(2\omega t)\big] & \text{for } n=2 \\[2mm] n\bar{Q}\,\frac{A_N B_N^2}{C}\,x & \text{for } n=3 \end{cases} \qquad (C.32)$$

$$F_{Ny} = \begin{cases} 2\bar{Q}\,\frac{A_N B_N^2}{C}\,\big[x\sin(2\omega t) + y(1-\cos(2\omega t))\big] & \text{for } n=2 \\[2mm] n\bar{Q}\,\frac{A_N B_N^2}{C}\,y & \text{for } n=3 \end{cases} \qquad (C.33)$$

$$F_{Sx} = \begin{cases} 2\bar{Q}\,\frac{A_S B_S^2}{C}\,\big[x(1-\cos(2\omega t)) - y\sin(2\omega t)\big] & \text{for } n=2 \\[2mm] n\bar{Q}\,\frac{A_S B_S^2}{C}\,x & \text{for } n=3 \end{cases} \qquad (C.34)$$

$$F_{Sy} = \begin{cases} 2\bar{Q}\,\frac{A_S B_S^2}{C}\,\big[-x\sin(2\omega t) + y(1+\cos(2\omega t))\big] & \text{for } n=2 \\[2mm] n\bar{Q}\,\frac{A_S B_S^2}{C}\,y & \text{for } n=3 \end{cases} \qquad (C.35)$$

These results are identical to the results obtained for the homopolar generator in Appendix I where it is shown how they are used in the stability and the response calculations.

Magnetic Forces at Cylindrical Airgap g_1

The total magnetic flux through the airgap g_1 is φ_1, (see Fig. 10) and the total reluctance is R_1; they are respectively given by eqs. (C.4) and (C.9). If we use the concept of permeance which is the inverse of reluctance, then:

$$P_1 = \frac{1}{R_1} = \frac{\mu L_1 2\pi r}{C_1} \qquad \text{(C.36)}$$

For a differential element $rd\theta$, the permeance is:

$$dP_1 = \frac{\mu L_1 \, rd\theta}{C_1 [1-\varepsilon_1 \cos(\theta-\alpha_1)]} \qquad \text{(C.37)}$$

Let the flux passing through $rd\theta$ be $d\varphi_1$. Then, from the magnetic circuit:

$$\frac{d\varphi_1}{dP_1} = \frac{\varphi_1}{P_1} = \frac{\varphi_1 C_1}{\mu L_1 2\pi r}$$

or

$$d\varphi_1 = \frac{1+\varepsilon_1 \cos(\theta-\alpha_1)}{2\pi} \varphi_1 \, d\theta \qquad \text{(C.38)}$$

Let B_1 be the flux density:

$$B_1 = \frac{d\varphi_1}{L_1 rd\theta} = \frac{1+\varepsilon_1 \cos(\theta-\alpha_1)}{L_1 2\pi r} \qquad \text{(C.39)}$$

Thus, the x-component of the magnetic forces is:

$$F_{1x} = \bar{Q} \int_0^{2\pi} B_1^2 \cos\theta \, L_1 rd\theta = \bar{Q} \frac{\varphi_1^2}{L_1 r (2\pi)^2} \int_0^{2\pi} [1+2\varepsilon_1 \cos(\theta-\alpha_1)] \cos\theta \, d\theta$$

$$= \bar{Q} \frac{\varepsilon_1 \varphi_1^2}{L_1 2\pi r} \cos\alpha = \bar{Q} \frac{\varphi_1^2}{2\pi r L_1 C_1} x_1 \qquad \text{(C.40)}$$

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Similarly, the y-component is:

$$F_{1y} = \bar{Q} \int_0^{2\pi} B_1^2 \sin\theta \, L_1 \, r \, d\theta = \bar{Q} \, \frac{\varepsilon_1 \varphi_1^2}{L_1 2\pi r} \sin\alpha_1 = \bar{Q} \, \frac{\varphi_1^2}{2\pi r L_1 C_1} \, y_1 \qquad (C.41)$$

where

$$x_1 = e_1 \cos\alpha_1 = C_1 \varepsilon_1 \cos\alpha_1$$

$$\qquad (C.42)$$

$$y_1 = e_1 \sin\alpha_1 = C_1 \varepsilon_1 \sin\alpha_1$$

Magnetic Forces at Conical Airgap g_2

The geometry of the conical airgap is shown in the diagram below.
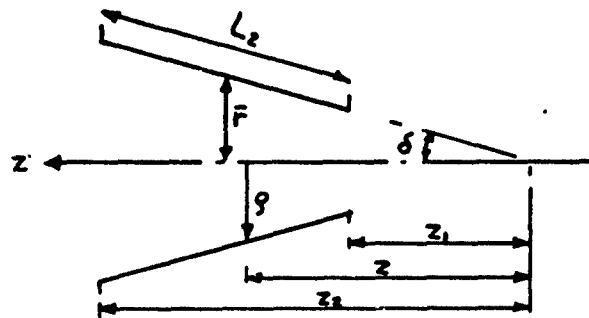
Figure 13: Geometry of Conical Airgap

$$g = z \cdot \tan\delta \qquad (C.43)$$

for a small element ($g \, d\theta \, \dfrac{dz}{\cos\delta}$), the permeance is:

$$dP_2 = \frac{\mu \, g \, d\theta \, \, dz/\cos\delta}{C_2 \left[1 - \varepsilon_2 \cos(\theta - \alpha_2) \right]} \qquad (C.44)$$

The subscript "2" is for airgap g_2. From the magnetic circuit, the flux through the small element is

$$\frac{d\varphi_2}{dP_2} = \frac{\varphi_2}{P_2} \qquad (C.45)$$

where $\quad \varphi_2 = \varphi_1 \qquad$ (see Fig. 10)

and $\quad P_2 = \frac{1}{R_2} \qquad$ (use eq. (C.11))

Thus

$$d\varphi_2 = \varphi_1 \frac{C_2}{\mu L_2 2\pi \bar{F}} \cdot \frac{\mu g \, d\theta \, dz/\cos\delta}{C_2} \left[1 + \varepsilon_2 \cos(\theta - \alpha_2)\right]$$

$$B_2 = flux\ density = \frac{d\varphi_2}{g \, d\theta \, dz/\cos\delta} = \frac{\varphi_1}{L_2 2\pi \bar{F}} \left[1 + \varepsilon_2 \cos(\theta - \alpha_2)\right] \qquad (C.46)$$

The corresponding x-component of the magnetic force becomes:

$$F_{2x} = \bar{Q} \int_0^{2\pi} \int_{z_1}^{z_2} B_2^2 \cos\theta \, g \, d\theta \, \frac{dz}{\cos\delta} \cos\delta = \bar{Q} \int_0^{2\pi} \int_{z_1}^{z_2} \left(\frac{\varphi_1}{L_2 2\pi \bar{F}}\right)^2 \left[1 + 2\varepsilon_2 \cos(\theta - \alpha_2)\right] \cos\theta \, d\theta \, z \tan\delta \, dz$$

$$= \bar{Q} \left(\frac{\varphi_1}{L_2 2\pi \bar{F}}\right)^2 2\varepsilon_2 \pi \cos\alpha_2 \tan\delta \int_{z_1}^{z_2} z \, dz \qquad (C.47)$$

But : $\quad \tan\delta \int_{z_1}^{z_2} z \, dz = (\tan\delta)\frac{1}{2}(z_2 + z_1)(z_2 - z_1) = (\tan\delta)\frac{1}{2}(z_2 + z_1) L_2 \cos\delta = \bar{F} L_2 \cos\delta$

i.e.:
$$F_{2x} = \bar{Q} \frac{\varphi_1^2}{L_2 2\pi \bar{F}} \varepsilon_2 \cos\delta \cos\alpha_2 = \bar{Q} \frac{\varphi_1^2 \cos\delta}{2\pi \bar{F} L_2 C_2} X_2 \qquad (C.48$$

Similarly,

$$F_{2y} = \bar{Q} \frac{\varphi_1^2}{L_2 2\pi \bar{F}} \varepsilon_2 \cos\delta \sin\alpha_2 = \bar{Q} \frac{\varphi_1^2 \cos\delta}{2\pi \bar{F} L_2 C_2} Y_2 \qquad (C.49)$$

where:
$$X_2 = C_2 \varepsilon_2 \cos\alpha_2$$
$$Y_2 = C_2 \varepsilon_2 \sin\alpha_2 \qquad (C.50)$$

For gap "3" and gap "4", we have by the same procedure,

$$F_{3x} = \bar{Q} \ \frac{\varphi_3^2}{L_1 2\pi r} \ \varepsilon_3 \cos a_3 = \bar{Q} \ \frac{\varphi_3^2}{2\pi r L_1 C_3} \ x_3 \tag{C.51}$$

$$F_{3y} = \bar{Q} \ \frac{\varphi_3^2}{L_1 2\pi r} \ \varepsilon_3 \sin a_3 = \bar{Q} \ \frac{\varphi_3^2}{2\pi r L_1 C_3} \ y_3 \tag{C.52}$$

$$F_{4x} = \bar{Q} \ \frac{\varphi_3^2}{L_2 2\pi \bar{r}} \ \cos\delta \ \varepsilon_4 \cos a_4 = \bar{Q} \ \frac{\varphi_3^2 \cos\delta}{2\pi \bar{r} L_2 C_4} \ x_4 \tag{C.53}$$

$$F_{4y} = \bar{Q} \ \frac{\varphi_3^2}{L_2 2\pi \bar{r}} \ \cos\delta \ \varepsilon_4 \sin a_4 = \bar{Q} \ \frac{\varphi_3^2 \cos\delta}{2\pi \bar{r} L_2 C_4} \ y_4 \tag{C.54}$$

Thus, the forces F_{1x} to F_{4y} can be represented by simple negative springs in the rotor response and rotor stability calculations.

In this appendix the magnetic field produced by the armature reaction will be studied. Consider Fig. 14 where the poles of the rotor move to the right:
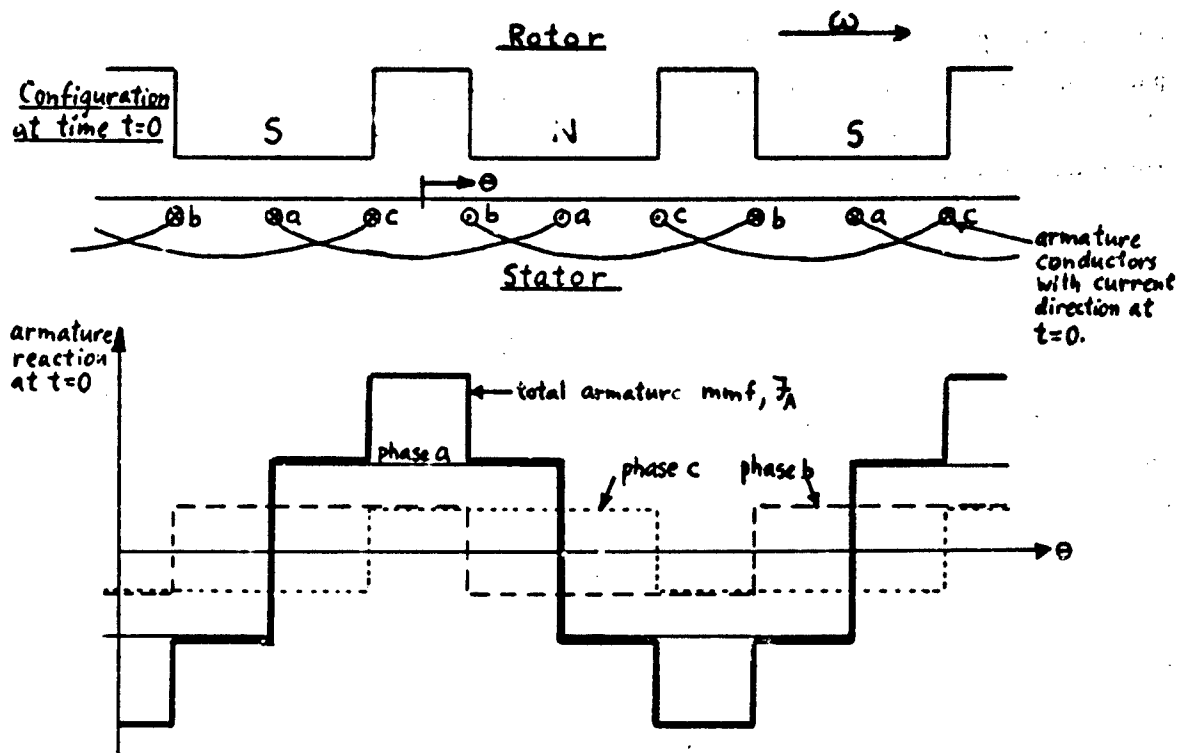
Figure 14: Armature Current Directions and Instantaneous mmf at t = o

This figure shows the rotor-stator position, the armature current directions and the instantaneous armature mmf at time t=0. The convention for current directions are: $\otimes$ means "into the paper" and $\odot$ means "out of the paper". To arrive at Figure D.1, assume that the airgaps at all the poles are the same. With 2n poles (n north poles and n south poles), the pole spacing becomes $\frac{\pi}{n}$ whereby the flux passing through the armature coils is:

for coil "a" $\varphi_a = i\, \varphi\, e^{in\omega t}$ (D.1)

for coil "b" $\varphi_b = i\, \varphi\, e^{i(n\omega t - \frac{2\pi}{3})}$ (D.2)

for coil "c" $\varphi_c = i\, \varphi\, e^{i(n\omega t - \frac{4\pi}{3})}$ (D.3)

where:

$$i = \sqrt{-1}$$

and all the φ's are complex numbers. The convention is that only the real part applies. To illustrate, assume that the flux φ is generated solely by the field coils. Then φ is real (i.e. there is no phase shift between the rotor motion and the flux) and eqs. (D.1) to (D.3) can be written as:

$$\varphi_a = Re\left\{i\varphi e^{in\omega t}\right\} = \varphi\, Re\left\{i(\cos(n\omega t)+i\sin(n\omega t))\right\} = -\varphi\sin(n\omega t),$$

$$\varphi_b = Re\left\{i\varphi e^{i(n\omega t - \frac{2\pi}{3})}\right\} = -\varphi\sin\left(n\omega t - \frac{2\pi}{3}\right) \qquad \text{(D.4)}$$

$$\varphi_c = Re\left\{i\varphi_c^{i(n\omega t - \frac{4\pi}{3})}\right\} = -\varphi\sin\left(n\omega t - \frac{4\pi}{3}\right)$$

Under load φ will lag the rotor motion as discussed later in which case a phase angle is introduced in eqs. (D.4).

Let each armature coil have N_A turns. The the induced voltage is

for coil "a" $\qquad e_a = -N_A \dfrac{d\varphi_a}{dt} = n\omega N_A \varphi$ $\qquad\qquad$ (D.5)

for coil "b" $\qquad e_b = n\omega N_A \varphi\, e^{-i\frac{2\pi}{3}}$ $\qquad\qquad$ (D.6)

for coil "c" $\qquad e_c = n\omega N_A \varphi\, e^{-i\frac{4\pi}{3}}$ $\qquad\qquad$ (D.7)

where the exponential $e^{in\omega t}$ has been left out for simplification. This will also be done in the following but it must always be recalled that properly this factor belongs in the equations.

The corresponding armature currents are:

for coil "a" $\qquad i_a = \dfrac{e_a}{Z_A} = \dfrac{n\omega N_A}{Z_A}\varphi$ $\qquad\qquad$ (D.8)

and similarly for the other two coils, where Z_A is the impedance of the output circuit. Setting t=0 in the above equations and using the right hand rule, the current directions in the armature conductors come out as shown in Figure 14. The corresponding mmf of the armature coils (the armature reaction) is found as:

$$\text{for coil "a"} \qquad mmf_a = N_A\, i_a = \frac{n\omega N_A^2}{Z_A}\, \varphi \qquad\qquad (D.9)$$

and similarly for coils "b" and "c". Hence, at time t=0 the armature mmf's are distributed around the circumference of the stator as a function of the angular coordinate Θ as shown in Figure 14. If they be represented by their fundamental harmonics instead of the "rectangular waves" shown in Figure 14, the mmf's of the three phases become:

$$\text{for phase "a"} \qquad \mathcal{F}_a = \frac{4}{\pi}\, mmf_a \cdot \cos(n\Theta) \qquad\qquad (D.10)$$

$$\text{for phase "b"} \qquad \mathcal{F}_b = \frac{4}{\pi}\, mmf_b \cdot \cos\!\left(n\Theta - \frac{2\pi}{3}\right) \qquad\qquad (D.11)$$

$$\text{for phase "c"} \qquad \mathcal{F}_c = \frac{4}{\pi}\, mmf_c \cdot \cos\!\left(n\Theta - \frac{4\pi}{3}\right) \qquad\qquad (D.12)$$

where the factor $\frac{4}{\pi}$ derives from taking the first harmonic of a rectangular wave. Here the origin for the angle Θ is between two poles at t=0 as shown in Figure 14.

To find the total armature mmf, $\mathcal{F}_A$, the mmf's of the three phases must be added:

$$\mathcal{F}_A = \mathcal{F}_a + \mathcal{F}_b + \mathcal{F}_c = \frac{4}{\pi}\frac{n\omega N_A^2}{Z_A}\,\varphi\left\{\cos(n\Theta) + e^{-i\frac{2\pi}{3}}\cos\!\left(n\Theta - \frac{2\pi}{3}\right) + e^{-i\frac{4\pi}{3}}\cos\!\left(n\Theta - \frac{4\pi}{3}\right)\right\}e^{in\omega t}$$

$$= \frac{4}{\pi}\frac{n\omega N_A^2}{Z_A}\,\varphi \cdot \frac{3}{2}\left[\cos(n\Theta) - i\sin(n\Theta)\right]e^{in\omega t} = \frac{6}{\pi}\frac{n\omega N_A^2}{Z_A}\,\varphi\, e^{in(\omega t - \Theta)} \qquad (D.13)$$

This represents a wave travelling synchronous with the rotor.

Let the load be balanced and let the effective value or ammeter value of the

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current per phase be I_A. Then, from eq. (D.8):

$$I_A = \frac{|i_a|}{\sqrt{2}} = \frac{n \omega N_A}{\sqrt{2}} \frac{|\varphi|}{|Z_A|} \tag{D.14}$$

Similarly, let the effective value or voltmeter value of the line voltage per phase be E_A so that from eq. (D.5):

$$E_A = \frac{|e_a|}{\sqrt{2}} = \frac{n \omega N_A}{\sqrt{2}} |\varphi| \tag{D.15}$$

Introduce the power factor angle ψ:

$$\text{power factor} = \cos \psi = \frac{Re\{Z_A\}}{|Z_A|} \qquad\qquad \sin \psi = \frac{Im\{Z_A\}}{|Z_A|} \tag{D.16}$$

which means:

$$\frac{1}{Z_A} = \frac{1}{|Z_A|} (\cos \psi - i \sin \psi) = \frac{1}{|Z_A|} e^{-i\psi} \tag{D.17}$$

The physical interpretation of the power factor angle ψ can be obtained by substituting eq. (D.17) into eq. (D.8):

$$i_a = \frac{e_a}{Z_A} = \frac{e_a}{|Z_A|} e^{-i\psi} \tag{D.18}$$

or, in other words, ψ gives the phase angle by which the armature current lags the voltage.

Next, introduce the power angle δ by the equations:

$$\cos \delta = \frac{Re\{\varphi\}}{|\varphi|} \qquad\qquad \sin \delta = \frac{-Im\{\varphi\}}{|\varphi|} \tag{D.19}$$

or:

$$\varphi = |\varphi| \cdot (\cos \delta - i \sin \delta) = |\varphi| e^{-i\delta} \tag{D.20}$$

Hence, δ gives the angle by which the flux lags the rotor rotation. It is called the power angle since it is the angle the rotor must pull ahead of the resultant magnetic field to supply the required load. The angle can be measured actually on the generator by means of a stroboscope. If the physical angle is measured as δ', then:

$$\delta = n\delta'$$

(D.21)

since the electrical frequency is n times greater than the mechanical rotational frequency.

In order to understand the relationship between the armature mmf and the airgap flux ϕ, it is useful to consider the phasor diagram of the alternator where each of the above quantities are taken as vectors rotating with the angular speed $(n\omega)$. Comparing eqs. (D.1) and (D.5) it is seen that the line voltage lags the flux ϕ by 90 degrees. The total mmf, $\mathcal{F}$, required to produce this flux is, of course, in phase with the flux. The armature reaction, $\mathcal{F}_A$, is in phase with the armature current (eq. (D.9)) which, as discussed above, lags the line voltage by the power factor angle ψ:
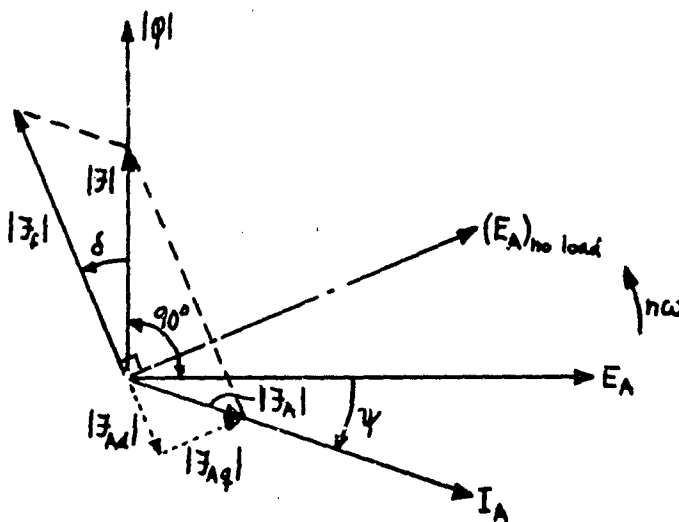
Figure 15: Phasor Diagram

The total mmf, $\vec{3}$, is the vector sum of the field mmf, $\vec{3}_f$, and the armature reaction:

$$\vec{3} = \vec{3}_f + \vec{3}_A \tag{D.22}$$

Thus, to offset $\vec{3}_A$ and meet the required total mmf, the field mmf, $\vec{3}_f$, must lead the flux φ . The lead angle is the power angle δ as discussed above.

To express the armature reaction $\vec{3}_A$, substitute eqs. (D.17) and (D.20) into eq. (D.13):

$$\vec{3}_A = \frac{6}{\pi} n\omega N_A^2 \frac{|\varphi|}{|\vec{z}_A|} e^{i(n(\omega t - \theta) - \psi - \delta)} \tag{D.23}$$

or by introducing the effective value of the armature current, I_A , from eq. (D.14):

$$\vec{3}_A = \frac{6\sqrt{2}}{\pi} N_A I_A \cos(n(\omega t - \theta) - \psi - \delta) \tag{D.24}$$

In practice, windings are fractionally pitched and distributed to provide better design (efficiency, wave form and winding configuration). This produces a spatial phase angle between the conductors of a given coil (or winding) so that the mmf per phase is reduced slightly from the value it would have for a concentrated, full pitch winding. Hence, eq. (D.24) is modified to:

$$\vec{3}_A = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(n(\omega t - \theta) - \psi - \delta) \tag{D.25}$$

where

$$K_d = distribution \ factor \quad (K_d \leq 1) \tag{D.26}$$

$$K_p = pitch \ factor \quad (K_p \leq 1) \tag{D.27}$$

$K_d = 1$ for a concentrated winding and may be taken as $K_d \approx 0.9$ for a distributed winding. $K_p = 1$ for a full pitch winding and $K_p = 0.9$ for a 5/6 pitch winding.

The armature reaction can be decomposed into a de-magnetizing component $\mathcal{F}_{Ad}$ in line with the field mmf, $\mathcal{F}_f$, and a cross-magnetizing component $\mathcal{F}_{Aq}$ lagging $\mathcal{F}_f$ by 90 degrees. From eq. (D.25):

$$\mathcal{F}_A = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \left[\cos(\psi+\delta)\cos(n(\omega t - \theta)) + \sin(\psi+\delta)\sin(n(\omega t - \theta)) \right] = \mathcal{F}_{Aq} + \mathcal{F}_{Ad} \qquad (D.28)$$

i.e.

$$\mathcal{F}_{Aq} = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(\psi+\delta)\cdot \cos(n(\omega t - \theta)) \qquad (D.29)$$

$$\mathcal{F}_{Ad} = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \sin(\psi+\delta)\cdot \sin(n(\omega t - \theta)) \qquad (D.30)$$

or:

$$\left| \mathcal{F}_{Aq} \right| = \mathcal{F}_A \cos(\psi+\delta) = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(\psi+\delta) \qquad (D.31)$$

$$\left| \mathcal{F}_{Ad} \right| = \mathcal{F}_A \sin(\psi+\delta) = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \sin(\psi+\delta) \qquad (D.32)$$

as readily seen from the phasor diagram in Figure D.2.

With these results, the circumferential distribution of mmf can be shown schematically as (see also Figure 14):
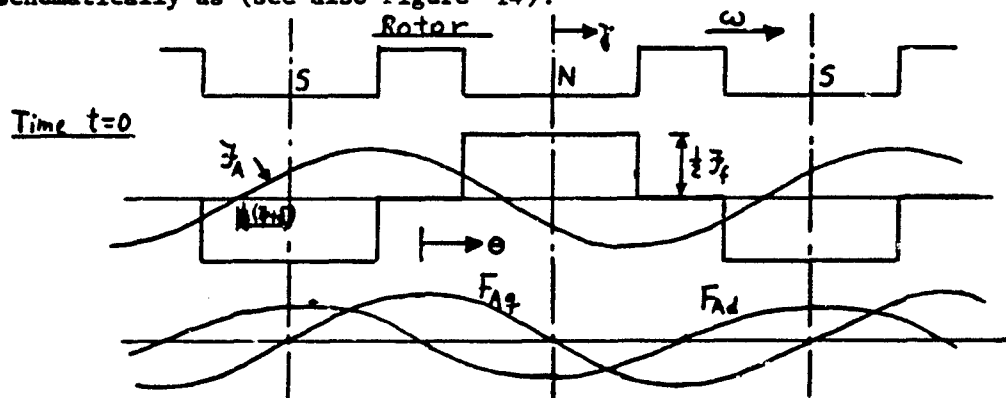
Figure 16: Circumferential mmf Distribution

The total field mmf is distributed evenly among the poles such that the field contributes $\frac{1}{2}\mathfrak{F}_f$ to each pole airgap (see Appendix I). This field follows the rotor in its rotation. Thus, if instead as in the foregoing where the armature reaction has been expressed as a function of the stationary coordinate θ, it is instead "seen" from the rotor, the angular coordinate γ, fixed in the rotor becomes (see Figure 16):

$$\gamma + \frac{\pi}{2n} = \theta - \omega t \qquad (D.33)$$

such that eqs. (D.29) and (D.30) yield:

$$\mathfrak{F}_{Aq} = -|\mathfrak{F}_{Aq}| \sin(n\gamma) \qquad (D.34)$$

$$\mathfrak{F}_{Ad} = -|\mathfrak{F}_{Ad}| \cos(n\gamma) \qquad (D.35)$$

where $|\mathfrak{F}_{Aq}|$ and $|\mathfrak{F}_{Ad}|$ are given by eqs. (D.31) and (D.32). It is seen that the de-magnetizing component, as the name implies, opposes the field mmf except for the unlikely case where $(\psi + \delta) < 0$.

APPENDIX V: Magnetic Forces of a Homopolar Generator - Modifications due to
Generator Load

In Appendix I the magnetic forces of a homopolar generator operating under no
load have been derived. In this appendix the effect of load will be investigated.

In Appendix IV it is shown that the mmf across the airgap at any pole is made up
of three components: $\frac{1}{2} \mathcal{F}_f$ ($\mathcal{F}_f$ is the mmf of the single field coil), the
de-magnetizing component $\mathcal{F}_{Ad}$ and the cross-magnetizing component $\mathcal{F}_{Aq}$,
where the latter two components make up the armature reaction (see Figure 16
Appendix IV). Thus, with the angular coordinate γ fixed in the rotor
and measured from a pole centerline, the airgap mmf at the north poles and at the
southpoles can be written:

$$\mathcal{F}_N = \mathcal{F}_S = \frac{1}{2} \mathcal{F}_f \left[1 - f_d \cos(n\gamma) - f_q \sin(n\gamma) \right] \qquad (E.1)$$

where:

$$f_d = \frac{|\mathcal{F}_{Ad}|}{\frac{1}{2}\mathcal{F}_f} = \frac{2}{\mathcal{F}_f} \cdot \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \sin(\psi + \delta) \qquad (E.2)$$

$$f_q = \frac{|\mathcal{F}_{Aq}|}{\frac{1}{2}\mathcal{F}_f} = \frac{2}{\mathcal{F}_f} \cdot \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(\psi + \delta) \qquad (E.3)$$

(see eqs. (D.31) and (D.32), and eqs. (D.34) and (D.35)).

Consider first the north poles. There are n northpoles and the airgap at the
k'th pole becomes:

$$\underline{k = 1, 2, \cdots, n} \qquad h_{Nk} = C \left[1 - \varepsilon \cos\left(\omega t - \alpha + \frac{2\pi}{n}(k-1) + \gamma\right) \right] \qquad (E.4)$$

where ω is the angular speed, C the radial clearance, α is the attitude angle
and ε is the eccentricity ratio. The equation is derived in Appendix I,
eq. (A.11), the only difference being the inclusion of the angle γ to take
into account the variation in airgap along the pole face. The rotor displacement

. in the plane of the northpoles are:

$$x_N = C_\varepsilon \cos\alpha$$

$$y_N = C_\varepsilon \sin\alpha$$

(E.5)

The flux density B_{Nk} is given by:

$$B_{Nk} = \frac{\mu \mathfrak{I}_N}{h_{Nk}} = B_0 \frac{[1 - f_d \cos(n\gamma) - f_q \sin(n\gamma)]}{[1 - \varepsilon \cdot \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1))]}$$

(E.6)

where:

$$B_0 = \frac{\mu \mathfrak{I}_f}{2C}$$

(E.7)

B_0 represents the average flux density if there is no generator load (see eq. (A.31), Appendix I).

When the pole length is ℓ , the radius of the pole face is r, and the pole extends over an angle β , the radial force pulling on the rotor at the k'th pole becomes:

$$radial \quad force = \bar{Q} r \ell \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} B_{Nk}^2 \, d\gamma$$

(E.8)

where $\bar{Q}$ is a numerical conversion factor. The corresponding x and y components of the force are:

$$\left.\begin{matrix} (F_{Nx})_k \\ (F_{Ny})_k \end{matrix}\right\} = \bar{Q} r \ell \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} B_{Nk}^2 \left\{\begin{matrix} \cos\left(\omega t + \frac{2\pi}{n}(k-1) + \gamma\right) \\ \sin\left(\omega t + \frac{2\pi}{n}(k-1) + \gamma\right) \end{matrix}\right\} d\gamma$$

(E.9)

Since the eccentricity ratio $\varepsilon << 1$, B_{Nk}^2 can be found from eqs. (E.6) and (E.5) as:

$$B_{Nk}^2 = \frac{B_0^2}{C}\left[1 - f_d \cos(n\gamma) - f_q \sin(n\gamma)\right]^2 \left[1 + 2x_N \cos\left(\omega t + \frac{2\pi}{n}(k-1) + \gamma\right) + 2y_N \sin\left(\omega t + \frac{2\pi}{n}(k-1) + \gamma\right)\right]$$ (E.10)

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where:

$$[1-f_d\cos(n\eta)-f_q\sin(n\eta)]^2 = 1+\tfrac{1}{2}f_d^2+\tfrac{1}{2}f_q^2-2f_d\cos(n\eta)-2f_q\sin(n\eta)+\tfrac{1}{2}(f_d^2-f_q^2)\cos(2n\eta)+f_d f_q\sin(2n\eta) \quad (E.11)$$

For simplification, set:

$$\eta = \omega t + \tfrac{2\pi}{n}(k-1)$$

Then, by substitution of eq. (E.10) into eq. (E.9):

$$\left.\begin{array}{c}(F_{Nx})_k\\[2ex](F_{Ny})_k\end{array}\right\} = \bar{Q}\,\frac{B_0^2\, r\ell}{C}\int_{-\frac{\theta}{2}}^{\frac{\theta}{2}}[1-f_d\cos(n\eta)-f_q\sin(n\eta)]^2\left\{\begin{array}{l}[\cos(\eta+\gamma)+x_N(1+\cos2(\eta+\gamma))+y_N\sin2(\eta+\gamma)]\\[2ex][\sin(\eta+\gamma)+x_N\sin2(\eta+\gamma)+y_N(1-\cos2(\eta+\gamma))]\end{array}\right\}d\gamma \quad (E.12)$$

The total forces acting on the rotor in the plane of the north poles are:

$$F_{Nx} = \sum_{k=1}^{n}(F_{Nx})_k$$

$$F_{Ny} = \sum_{k=1}^{n}(F_{Ny})_k \tag{E.13}$$

As shown in Appendix I:

$$\sum_{k=1}^{n}\cos\left(k\,\tfrac{2\pi}{n}\right) = \sum_{k=1}^{n}\sin\left(k\,\tfrac{2\pi}{n}\right) = \sum_{k=1}^{n}\sin\left(k\,\tfrac{4\pi}{n}\right) = 0 \tag{E.14}$$

$$\sum_{k=1}^{n}\cos\left(k\,\tfrac{4\pi}{n}\right) = \begin{cases}2 & \text{for } n=2\\[1ex]0 & \text{for } n\geq 3\end{cases} \tag{E.15}$$

Hence:

$$\sum_{k=1}^{n}\cos\eta = \sum_{k=1}^{n}\sin\eta = 0 \tag{E.16}$$

$$\sum_{k=1}^{n}\cos2\eta = \begin{cases}2\cos(2\omega t) & \text{for } n=2\\[1ex]0 & \text{for } n\geq 3\end{cases} \tag{E.17}$$

$$\sum_{k=1}^{n}\sin2\eta = \begin{cases}2\sin(2\omega t) & \text{for } n=2\\[1ex]0 & \text{for } n\geq 3\end{cases} \tag{E.18}$$

Therefore, for n=2 eq. (E.12) may be summed to give:

$$\left.\begin{matrix} F_{Nx} \\ \\ F_{Ny} \end{matrix}\right\} = \bar{Q}\,\frac{B_o^2 r^2}{C} \left\{ \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1-f_d \cos(n\gamma)-f_q \sin(n\gamma)\right]^2 \left\{ \begin{matrix} nx_N+2(x_N \cos2\omega t+y_N \sin2\omega t)\cos2\gamma-2(x_N \sin2\omega t-y_N \cos2\omega t)\sin2\gamma \\ \\ ny_N+2(x_N \sin2\omega t-y_N \cos2\omega t)\cos2\gamma+2(x_N \cos2\omega t+y_N \sin2\omega t)\sin2\gamma \end{matrix} \right\} d\gamma \right. \tag{E.19}$$

Substituting from eq. (E.11), the following integrals can be evaluated:

$$I_o = \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1-f_d \cos(n\gamma)-f_q \sin(n\gamma)\right]^2 d\gamma = \left(1+\tfrac{1}{2}f_d^2+\tfrac{1}{2}f_q^2\right)\beta + \tfrac{1}{2n}\left(f_d^2-f_q^2\right)\sin(n\beta) - \tfrac{4}{n}f_d \sin\left(\tfrac{n\beta}{2}\right) \tag{E.20}$$

$$I_1 = \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1-f_d \cos(2\gamma)-f_q \sin(2\gamma)\right]^2 \cos2\gamma\, d\gamma = \left(1+\tfrac{1}{2}f_d^2+\tfrac{1}{2}f_q^2\right)\sin\beta + \tfrac{1}{4}\left(f_d^2-f_q^2\right)\left(\tfrac{1}{3}\sin3\beta+\sin\beta\right)-f_d\left(\beta+\tfrac{1}{2}\sin2\beta\right) \tag{E.21}$$

$$I_2 = -\int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1-f_d \cos(2\gamma)-f_q \sin(2\gamma)\right]^2 \sin2\gamma\, d\gamma = f_q\left(\beta-\tfrac{1}{2}\sin2\beta\right) + \tfrac{1}{2}f_d f_q\left(\tfrac{1}{3}\sin3\beta-\sin\beta\right) \tag{E.22}$$

In total, eq. (E.19) then becomes:

for n=2

$$F_{Nx} = 2\bar{Q}\,\frac{B_o^2 r\ell}{C}\left[\left(I_o+I_1\cos(2\omega t)+I_2\sin(2\omega t)\right)x_N + \left(-I_2\cos(2\omega t)+I_1\sin(2\omega t)\right)y_N\right] \tag{E.23}$$

$$F_{Ny} = 2\bar{Q}\,\frac{B_o^2 r\ell}{C}\left[\left(-I_2\cos(2\omega t)+I_1\sin(2\omega t)\right)x_N + \left(I_o-I_1\cos(2\omega t)-I_2\sin(2\omega t)\right)y_N\right] \tag{E.24}$$

for n=3

$$F_{Nx} = n\bar{Q}\,\frac{B_o^2 r\ell}{C}\,I_o x_N \tag{E.25}$$

$$F_{Ny} = n\bar{Q}\,\frac{B_o^2 r\ell}{C}\,I_o y_N \tag{E.26}$$

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In the case where the angular extension β of the pole is sufficiently small that $\sin\beta \cong \beta$ and $\sin(n\beta) \cong n\beta$ eqs. (E.20) to (E.22) reduce to:

for $\sin(n\beta) \cong n\beta$

$$I_0 = I_1 = \beta(1-f_d)^2$$

(E.27)

$$I_2 = 0$$

whereby eqs. (E.23) to (E.26) become:

for n=2

$$F_{Nx} = 2\bar{Q}\,\frac{B_0^2 A}{C}(1-f_d)^2\left[x_N(1+\cos(2\omega t)) + y_N\sin(2\omega t)\right]$$

(E.28)

$$F_{Ny} = 2\bar{Q}\,\frac{B_0^2 A}{C}(1-f_d)^2\left[x_N\sin(2\omega t) + y_N(1-\cos(2\omega t)\right]$$

(E.29)

for $n\geq 3$

$$F_{Nx} = n\bar{Q}\,\frac{B_0^2 A}{C}(1-f_d^2)\,x_N$$

(E.30)

$$F_{Ny} = n\bar{Q}\,\frac{B_0^2 A}{C}(1-f_d^2)\,y_N$$

(E.31)

where $A = \ell r\beta$ is the pole area. These equations are the same as for the no-load case investigated in Appendix I except for the factor $(1-f_d)^2$. In Appendix I is also given the information on preparing the corresponding input for the rotor stability and the rotor response programs.

Comparing eqs. (E.28) to (E.31) with eqs. (A.32) to (A.35) in Appendix I, it is seen that the armature reaction reduces the magnetic forces but has no other effect.

APPENDIX VI: Magnetic Forces of a Heteropolar Inductor Generator - Modifications Due to Generator Load

In Appendix II, the magnetic forces of a heteropolar generator have been derived for the case of no load on the generator. In the case where the generator is loaded, there will be an armature reaction in form of a reverse flux set up by the current in the armature coils. This reverse flux modifies the flux due to the field coils and, hence, modifies the magnetic forces.

The generator is shown schematically in Fig. 2. Consider first the k'th pole of the generator:
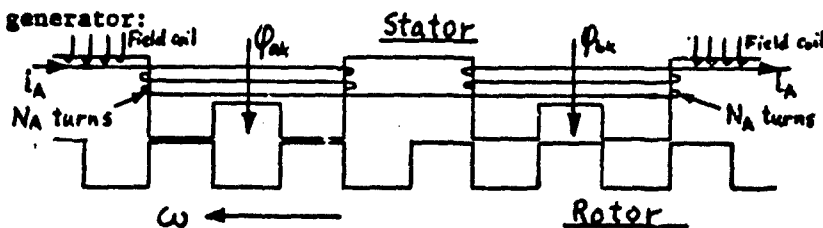
Figure 17: Windings of the k'th Pole of the Generator

The coil has two windings per pole as shown. The flux generated by the field coils passes the pole through the two windings such that the flux through one winding is φ_{ak} and the other winding φ_{bk}. If there are n_r rotor teeth in total, then the two flux components become:
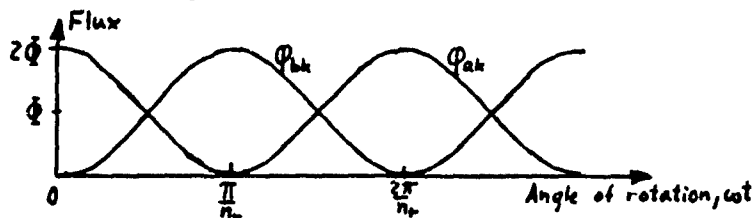
$$\varphi_{ak} = \bar{\Phi}\left[1 + \cos(\nu t)\right] \tag{F.1}$$

$$\varphi_{bk} = \bar{\Phi}\left[1 - \cos(\nu t)\right] \tag{F.2}$$

$$\nu = n_r\,\omega \tag{F.3}$$

Thus the electric frequency of the single phase output voltage is $\nu = n_r \omega$.
Let the generator be connected to an output circuit with impedance Z_A.
With n north poles and n south poles the circuit diagram is then:
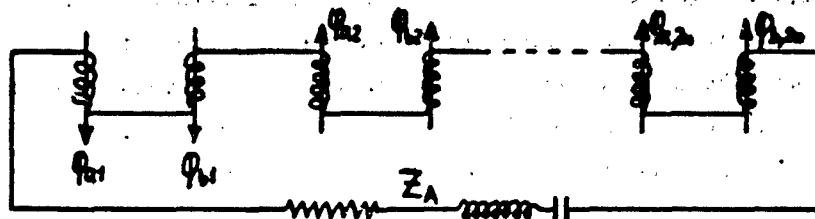
The current produced in the circuit is i_A which is determined from:

$$Z_A i_A + N_A \sum_1^{2n} \left[\frac{d\varphi_{ak}}{dt} - \frac{d\varphi_{bk}}{dt} \right] = 0 \qquad (F.4)$$

where N_A is the number of turns of one winding of an armature coil.

The reluctances of the airgaps at the k'th pole are $\mathcal{R}_{ak}$ and $\mathcal{R}_{bk}$ respectively. They are given by (see Appendix II):

$$\frac{1}{\mathcal{R}_{ak}} = (1 + \cos(\nu t)) \left[P + \varepsilon (G_k - H_k) \right] \qquad (F.5)$$

$$\frac{1}{\mathcal{R}_{bk}} = (1 - \cos(\nu t)) \left[P + \varepsilon (G_k + H_k) \right] \qquad (F.6)$$

where:

$$P = \frac{\mu A_T n_s}{2C} \qquad (F.7)$$

$$G_k = \frac{\mu A_T}{2C} \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right) \cdot \sum_{j=1}^{n_s} \cos\left((4j-1)\frac{\pi}{2n_r}\right) \qquad (F.8)$$

$$H_k = \frac{\mu A_T}{2C} \sin\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right) \cdot \sum_{j=1}^{n_s} \sin\left((4j-1)\frac{\pi}{2n_r}\right) \qquad (F.9)$$

where A_T is the area of a stator tooth, μ is the permeability of air, C is the radial airgap, n is the number of north poles (= number of south poles),

ε is the eccentricity ratio of the rotor with respect to the stator and α is the corresponding attitude angle. Furthermore, there are $2n_s$ stator teeth per pole.

When the mmf across the airgap is f, the flux for the two sections of the k'th pole becomes:

$$\varphi_{ak} = \frac{1}{R_{ak}} f = (1 + \cos(\nu t))[P + \varepsilon(G_k - H_k)] f \qquad (F.10)$$

$$\varphi_{bk} = \frac{1}{R_{bk}} f = (1 - \cos(\nu t))[P + \varepsilon(G_k + H_k)] f \qquad (F.11)$$

whereby:

$$\sum_1^{2n} \left(\frac{d\varphi_{ak}}{dt} - \frac{d\varphi_{bk}}{dt} \right) = 2 \frac{d}{dt} \left\{ \sum_1^{2n} \left[(P + \varepsilon G_k) \cos(\nu t) - \varepsilon H_k \right] f \right\} \qquad (F.12)$$

The following identities hold true:

$$\sum_{k=1}^{2n} \cos\left(\frac{\pi}{n} k\right) = 0$$
$$\sum_{k=1}^{2n} \sin\left(\frac{\pi}{n} k\right) = 0 \qquad (F.13)$$

from which follows:

$$\sum_{k=1}^{2n} G_k = \sum_{k=1}^{2n} H_k = 0 \qquad (F.14)$$

i.e.:

$$\sum_{k=1}^{2n} \left(\frac{d\varphi_{ak}}{dt} - \frac{d\varphi_{bk}}{dt} \right) = 4n P \frac{d}{dt} \left(f \cos(\nu t) \right) \qquad (F.15)$$

The mmf across a winding of an armature coil is $N_A i_A$ which can be found from eq. (F.4) by substitution from eq. (F.15):

$$N_A i_A = - \frac{N_A^2}{Z_A} 4n P \frac{d}{dt} \left(f \cos(\nu t) \right) \qquad (F.16)$$

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Since this mmf is independent of rotor eccentricity, the mmf across all poles will be the same and equal to $\frac{1}{2}\mathfrak{F}_f$ per pole where $\mathfrak{F}_f$ is the mmf of a field coil (see Appendix II). Thus, the mmf across the k'th pole can be expressed in terms of its components:

Figure 18
The Magnetic Circuit
for the k'th pole
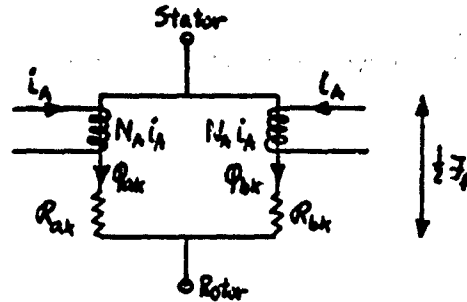
$$\tfrac{1}{2}\mathfrak{F}_f = N_A i_A + \mathcal{R}_{ak}\,\varphi_{ak} = N_A i_A + \mathcal{R}_{bk}\,\varphi_{bk} = N_A i_A + f = f - 4nP\frac{N_A^2}{Z_A}\frac{d}{dt}\left(f\cos\nu t\right) \quad (F.17)$$

Consider next the k'th field coil, located between the (k-1)'th and the k'th pole. The coil has N_f turns with a flux φ_k passing through it. The field coil circuit has an impressed d.c. voltage E_f, an impedance Z_f and a current i_f. Hence, the equation for the field coil is:

$$Z_f\, i_f + N_f\,\frac{d\varphi_k}{dt} = E_f \qquad\qquad (F.18)$$

from which the mmf of the field coil becomes:

$$\mathfrak{F}_f = N_f\, i_f = \frac{N_f}{R_f}\,E_f - \frac{N_f^2}{Z_f}\,\frac{d\varphi_k}{dt} \qquad\qquad (F.19)$$

where R_f is the resistance of the field coil circuit.

The total flux through the k'th pole is ($\varphi_{ak} + \varphi_{bk}$). Since the flux φ_k from each field coil passes through two poles, a summation over all field coils and all poles yields:

$$\varphi = \sum_{k=1}^{2n} \varphi_k = \tfrac{1}{2}\sum_{k=1}^{2n}\left(\varphi_{ak} + \varphi_{bk}\right) \qquad\qquad (F.20)$$

Hence, summing eq. (F.19) from k=1 to k=2n, the result becomes:

$$2n\,\mathfrak{F}_f = 2n\,\frac{N_f}{R_f}\,E_f - \frac{N_f^2}{Z_f}\sum_{k=1}^{2n}\frac{d\varphi_k}{dt} \qquad\qquad (F.21)$$

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or:

$$\mathfrak{Z}_f = \frac{N_f}{R_f} E_f - \frac{1}{4n} \frac{N_f^2}{Z_f} \frac{d}{dt} \left[\sum_{k=1}^{2n} (\varphi_{ak} + \varphi_{bk}) \right] \qquad (F.22)$$

Substitute for φ_{ak} and φ_{bk} from eqs. (F.10) and (F.11):

$$\varphi_{ak} + \varphi_{bk} = \left[2(P + \varepsilon G_k) - 2\varepsilon H_k \cos(\nu t) \right] f$$

or making use of eq. (F.14):

$$\sum_{k=1}^{2n} (\varphi_{ak} + \varphi_{bk}) = 4n P f \qquad (F.23)$$

whereby eq. (F.22) becomes:

$$\mathfrak{Z}_f = \frac{N_f}{R_f} E_f - P \frac{N_f^2}{Z_f} \frac{df}{dt} \qquad (F.24)$$

Equate eqs. (F.17) and (F.24) to get:

$$\mathfrak{Z}_f = \frac{N_f}{R_f} E_f - P \frac{N_f^2}{Z_f} \frac{df}{dt} = 2f - 8nP \frac{N_A^2}{Z_A} \frac{d}{dt} (f \cos(\nu t)) \qquad (F.25)$$

which is a first order, ordinary differential equation in the variable f. If Z_f and Z_A are pure resistances, this equation has a closed form solution which, however, is not to convenient for the present purposes anyway. Instead, f shall be expressed as a Fourier series:

$$f = f_0 \left[1 + \sum_{m=1}^{\infty} \left(f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi) \right) \right] \qquad (F.26)$$

where:

$$\psi = \nu t \qquad (F.27)$$

Hence:

$$f \cos(\nu t) = f \cos(\psi) = f_0 \left[\cos\psi + \tfrac{1}{2} \sum_{k=1}^{\infty} \left[f_{cm} \left(\cos(m+1)\psi + \cos(m-1)\psi \right) - f_{sm} \left(\sin(m+1)\psi + \sin(m-1)\psi \right) \right] \right] \qquad (F.28)$$

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Set:

$$f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi) = f_m e^{im\psi} \qquad (F.29)$$

where:

$$f_m = f_{cm} + i f_{sm} \qquad (F.30)$$
$$i = \sqrt{-1}$$

and it is understood that only the real part applies. Hence:

$$f = f_0 \left[1 + \sum_{m=1}^{\infty} f_m e^{i\psi} \right] \qquad (F.31)$$

$$f \cos(\nu t) = f_0 \left[e^{i\psi} + \tfrac{1}{2} \sum_{m=1}^{\infty} f_m \left(e^{i(m-1)\psi} + e^{i(m+1)\psi} \right) \right] \qquad (F.32)$$

Next, consider the impedances Z_A and Z_f :

$$\frac{1}{Z_A} \cong \frac{1}{R_A + i\left(m\nu L_A - \frac{1}{m\nu C_A}\right)} = Y_{A_m} \qquad (F.33)$$

$$\frac{1}{Z_f} \cong \frac{1}{R_f + i\left(m\nu L_f - \frac{1}{m\nu C_f}\right)} = Y_{f_m} \qquad (F.34)$$

where R is electrical resistance, C is capacitance and L is inductance (L does not include the field coils or the armature coils). Actually, the circuits may have more than one resonance but they can still be represented by frequency dependent impedances. Thus:

$$\frac{1}{Z_f} \frac{df}{dt} = i\nu f_0 \sum_{m=1}^{\infty} m Y_{f_m} f_m e^{im\psi} \qquad (F.35)$$

$$\frac{1}{Z_A} \frac{d}{dt}(f \cos\psi) = i\nu f_0 \left\{ Y_{A1} e^{i\psi} + \tfrac{1}{2} \sum_{m=1}^{\infty} f_m \left[(m-1) Y_{A,m-1} e^{i(m-1)\psi} + (m+1) Y_{A,m+1} e^{i(m+1)\psi} \right] \right\} \qquad (F.36)$$

Substituting eqs. (F.31), (F.35) and (F.36) into eq. (F.25) and collecting terms according to powers of $e^{im\psi}$, an infinite set of simultaneous equations are obtained:

$\underline{m=0}$ $\quad f_o = \frac{1}{2} N_f \frac{1}{R_f} E_f$ $\hfill$ (F.37)

$\underline{m=1}$ $\quad [2 + i\nu Y_{f_1} P N_f^2] f_1 - i4\nu Y_{A_1} n P N_A^2 f_2 = i8\nu Y_{A_1} n P N_A^2$ $\hfill$ (F.38)

$\underline{m \geq 2}$ $\quad -i4m\nu Y_{Am} n P N_A^2 f_{m-1} + [2 + im\nu Y_{fm} P N_f^2] f_m - i4m\nu Y_{Am} n P N_A^2 f_{m+1} = 0$ $\hfill$ (F.39)

Define:

$$\alpha_m = 2 + im\nu Y_{fm} P N_f^2 \hfill (F.40)$$

$$\lambda_m = i4m\nu Y_{Am} n P N_A^2 \hfill (F.41)$$

As $m \to \infty$, $im\nu Y_{fm}$ becomes $\frac{1}{L_f}$ and $im\nu Y_{Am}$ becomes $\frac{1}{L_A}$ (see eqs. (F.33) and (F.34)), i.e.:

$$\alpha_m \to 2 + \frac{P N_f^2}{L_f}$$

$$\lambda_m \to \frac{4n P N_A^2}{L_A} \hfill (F.42)$$

Substituting eqs. (F.40) and (F.41) into eq. (F.39):

$\underline{m \geq 2}$ $\qquad -\lambda_m f_{m-1} + \alpha_m f_m - \lambda_m f_{m+1} = 0$ $\hfill$ (F.43)

Assume, that eq. (F.42) reduces to identities for all $m \geq M$, and add all the corresponding eqs. (F.43) for $m \geq M$ to get:

$$(\alpha_M - 2\lambda_M) \sum_{m=M}^{\infty} f_m = \lambda_{M-1} f_{M-2} - (\alpha_{M-1} - \lambda_M) f_{M-1} \hfill (F.44)$$

from which it is concluded, that since the sum $\sum_{m=M}^{\infty} f_m$ which contains infinite many terms, has a finite value, each term f_m must be small. This is, of course, an inadequate proof from a mathematical point of view but is is sufficient from a physical point of view. Thus, all higher harmonics of f may be ignored but before solving for f, the magnetic forces acting on the rotor will

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be derived.

The magnetic forces are F_x and F_y. They have been determined in Appendix II for the case of no load where the mmf across the airgaps at the poles is equal to $\frac{1}{2}\mathfrak{Z}_0$ ($\mathfrak{Z}_0 = \frac{N_f}{R_f}E_f$). In the present case of a loaded generator this mmf is equal to f. Thus, substituting f for $\frac{1}{2}\mathfrak{Z}_0$ in eqs. (B.36) and (B.37) of Appendix II, the magnetic forces become:

$$F_x = \bar{Q}\,\frac{2nn_s A_T \mu^2}{C^3}\,f^2 x \qquad\qquad (F.45)$$

$$F_y = \bar{Q}\,\frac{2nn_s A_T \mu^2}{C^3}\,f^2 y \qquad\qquad (F.46)$$

The flux density B_0 in the case of the unloaded generator is:

$$B_0 = \frac{\mu\frac{1}{2}\mathfrak{Z}_0}{C} = \frac{\mu N_f E_f}{2C R_f} = \frac{\mu f_0}{C} \qquad\qquad (F.47)$$

whereby eqs. (F.45) and (F.46) can be written:

$$F_x = \bar{Q}\,\frac{2nn_s A_T B_0^2}{C}\left(\frac{f}{f_0}\right)^2 x \qquad\qquad (F.48)$$

$$F_y = \bar{Q}\,\frac{2nn_s A_T B_0^2}{C}\left(\frac{f}{f_0}\right)^2 y \qquad\qquad (F.49)$$

Here, $\frac{f}{f_0}$ is given by eq. (F.26):

$$\frac{f}{f_0} = 1 + \sum_{m=1}^{\infty}\left(f_{c_m}\cos(m\psi) - f_{s_m}\sin(m\psi)\right) \qquad\qquad (F.50)$$

from which:

$$\left(\frac{f}{f_0}\right)^2 = 1 + 2\sum_{m=1}^{\infty}\left(f_{c_m}\cos(m\psi) - f_{s_m}\sin(m\psi)\right) + \sum_{m=1}^{\infty}\left\{\left(f_{c_m}\cos(m\psi) - f_{s_m}\sin(m\psi)\right)\sum_{\ell=1}^{\infty}\left(f_{c_\ell}\cos(\ell\psi) - f_{s_\ell}\sin(\ell\psi)\right)\right\} \quad (F.51)$$

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Making use of the trigonometric identities:

$$\cos(m\psi)\cos(\ell\psi) = \tfrac{1}{2}\left[\cos(m+\ell)\psi + \cos(m-\ell)\psi\right]$$

$$\cos(m\psi)\sin(\ell\psi) = \tfrac{1}{2}\left[\sin(m+\ell)\psi - \sin(m-\ell)\psi\right]$$

$$\sin(m\psi)\cos(\ell\psi) = \tfrac{1}{2}\left[\sin(m+\ell)\psi + \sin(m-\ell)\psi\right]$$

(F.52)

$$\sin(m\psi)\sin(\ell\psi) = \tfrac{1}{2}\left[\cos(m-\ell)\psi - \cos(m+\ell)\psi\right]$$

eq. (F.51) can also be written:

$$\left(\frac{f}{f_0}\right)^2 = 1 + 2\sum_{m=1}^{\infty}\left(f_{cm}\cos(m\psi) - f_{sm}\sin(m\psi)\right) + \tfrac{1}{2}\sum_{m=1}^{\infty}\sum_{\ell=1}^{\infty}\Big\{ f_{cm}f_{c\ell}\left[\cos(m+\ell)\psi + \cos(m-\ell)\psi\right]$$
$$- f_{sm}f_{s\ell}\left[\cos(m+\ell)\psi - \cos(m-\ell)\psi\right] - f_{sm}f_{c\ell}\left[\sin(m+\ell)\psi + \sin(m-\ell)\psi\right] - f_{cm}f_{s\ell}\left[\sin(m+\ell)\psi - \sin(m-\ell)\psi\right]\Big\} \quad\text{(F.53)}$$

Collect terms $\sin\cos(m\psi)$ and $\sin(m\psi)$ to get:

$$\left(\frac{f}{f_0}\right)^2 = 1 + \tfrac{1}{2}\sum_{m=1}^{\infty}\left(f_{cm}^2 + f_{sm}^2\right) + \left[2f_{c1} + \sum_{m=1}^{\infty}\left(f_{cm}f_{c,m+1} + f_{sm}f_{s,m+1}\right)\right]\cos\psi$$

$$- \left[2f_{s1} + \sum_{m=1}^{\infty}\left(f_{cm}f_{s,m+1} - f_{sm}f_{c,m+1}\right)\right]\sin\psi$$

$$+ \sum_{m=2}^{\infty}\Big\{\left[2f_{cm} + \sum_{\ell=1}^{\infty}\left(f_{c\ell}f_{c,m+\ell} + f_{s\ell}f_{s,m+\ell}\right) + \tfrac{1}{2}\sum_{\ell=1}^{m-1}\left(f_{c\ell}f_{c,m-\ell} - f_{s\ell}f_{s,m-\ell}\right)\right]\cos(m\psi)$$

$$- \left[2f_{sm} + \sum_{\ell=1}^{\infty}\left(f_{c\ell}f_{s,m+\ell} - f_{s\ell}f_{c,m+\ell}\right) + \tfrac{1}{2}\sum_{\ell=1}^{m-1}\left(f_{c\ell}f_{s,m-\ell} + f_{s\ell}f_{c,m-\ell}\right)\right]\sin(m\psi)\Big\} \quad\text{(F.54)}$$

Substitution of this equation into eqs. (F.48) and (F.49) results in the final expressions for the magnetic forces.

Returning to the solution of eqs. (F.38) and (F.39), they define an infinite set of simultaneous equations

$$\begin{Bmatrix} \varkappa_1 & -\lambda_1 & 0 & 0 & 0 & ---- \\ -\lambda_2 & \varkappa_2 & -\lambda_2 & 0 & 0 & ---- \\ 0 & -\lambda_3 & \varkappa_3 & -\lambda_3 & 0 & ---- \\ 0 & 0 & -\lambda_4 & \varkappa_4 & -\lambda_4 & ---- \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{Bmatrix} \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ \vdots \end{Bmatrix} = \begin{Bmatrix} 2\lambda_1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{Bmatrix} \quad\text{(F.55)}$$

where $\varkappa_m$ and λ_m are complex numbers defined by eqs. (F.40) and (F.41). For $m \geq 2$ these equations are of the form given by eq. (F.43). Define δ_m by the equation:

$$f_m = \delta_{m-1} f_{m-1} \qquad (F.56)$$

and substitute into eq. (F.43) to get:

$$-\lambda_m f_{m-1} + (\varkappa_m - \lambda_m \delta_m) f_m = 0$$

Comparing this equation with eq. (F.56) yields:

$$\underline{m = 1,2,3,\cdots} \qquad \delta_{m-1} = \frac{\lambda_m}{\varkappa_m - \delta_m \lambda_m} \qquad (F.57)$$

Assume that all f_m for $m > M$ are so small that they can be ignored, i.e. $f_{M+1} = f_{M+2} = \cdots \cong 0$. Then, from eq. (F.56):

$$\delta_M = 0$$

With this as a starting condition, repeated use of eq. (F.57), starting with $m = M$ makes it possible to calculate $\delta_{M-1}, \delta_{M-2}, \cdots, \delta_1, \delta_0$. From the first equation of eq. (55):

$$\varkappa_1 f_1 - \lambda_1 f_2 = 2\lambda_1$$

one obtains:

$$f_1 = \frac{2\lambda_1}{\varkappa_1 - \delta_1 \lambda_1} = 2\delta_0 \qquad (F.58)$$

after which eq. (F.56 can be used to obtain the results for the other f-values:

$$\underline{m = 1,2,\cdots} \qquad f_m = 2\delta_0 \cdot \delta_1 \cdot \delta_2 \cdot \cdots \cdot \delta_{m-1} \qquad (F.59)$$

In the case where only the two first harmonics are important, the solution of eq. (55) can be found directly as:

$$f_1 = f_{c1} + i f_{s1} = \frac{2\lambda_1 \varkappa_2}{\varkappa_1 \varkappa_2 - \lambda_1 \lambda_2} \tag{F.60}$$

$$f_2 = f_{c2} + i f_{s2} = \frac{2\lambda_1 \lambda_2}{\varkappa_1 \varkappa_2 - \lambda_1 \lambda_2} \tag{F.61}$$

The same result can, of course, also be obtained by using the outlined general method. Here, $f_3 = f_4 = \cdots = 0$, so that M=2 and $\delta_2 = 0$. Then, from eq. (F.57):

$$\delta_1 = \frac{\lambda_1}{\varkappa_2}$$

$$\delta_0 = \frac{\lambda_1}{\varkappa_1 - \delta_1 \lambda_1} = \frac{\lambda_1 \varkappa_2}{\varkappa_1 \varkappa_2 - \lambda_1 \lambda_2}$$

i.e.:

$$f_1 = 2\delta_0 = \frac{2\lambda_1 \varkappa_2}{\varkappa_1 \varkappa_2 - \lambda_1 \lambda_2}$$

$$f_2 = 2\delta_0 \delta_1 = \frac{2\lambda_1 \lambda_2}{\varkappa_1 \varkappa_2 - \lambda_1 \lambda_2}$$

which agrees with eqs. (F.60) and (F.61).

Hence, by truncating the equations at m=2, the result becomes:

$$\frac{f}{f_0} = 1 + \left(f_{c1} \cos(\nu t) - f_{s1} \sin(\nu t) \right) + \left(f_{c2} \cos(2\nu t) - f_{s2} \sin(2\nu t) \right)$$

From eq. (F.54) with $f_{c3} = f_{s3} = f_{c4} = f_{s4} = \cdots = 0$:

$$\left(\frac{f}{f_0}\right)^2 = 1 + \tfrac{1}{2}\left(f_{c1}^2 + f_{s1}^2 + f_{c2}^2 + f_{s2}^2 \right) + \left[2 f_{c1} + f_{c1} f_{c2} + f_{s1} f_{s2} \right] \cos(\nu t)$$
$$- \left[2 f_{s1} + f_{c1} f_{s2} - f_{s1} f_{c2} \right] \sin(\nu t) + \left[2 f_{c2} + \tfrac{1}{2} f_{c1}^2 - \tfrac{1}{2} f_{s1}^2 \right] \cos(2\nu t)$$
$$- \left[2 f_{s2} + f_{c1} f_{s1} \right] \sin(2\nu t) + \left[f_{c1} f_{c2} - f_{s1} f_{s2} \right] \cos(3\nu t) - \left[f_{c1} f_{s2} + f_{s1} f_{c2} \right] \sin(3\nu t)$$
$$+ \tfrac{1}{2} \left[f_{c2}^2 - f_{s2}^2 \right] \cos(4\nu t) - f_{c2} f_{s2} \sin(4\nu t) \tag{F.62}$$

which then can be substituted into eqs. (F.48) and (F.49) to obtain the magnetic forces.

The response and stability computer programs consider only one frequency component of the magnetic forces. Let this be the first harmonic so that the magnetic force frequency Ω is:

$$\Omega = \nu = n_r \omega \tag{F.63}$$

i.e.

$$\frac{\Omega}{\omega} = n_r \tag{F.64}$$

Writing the magnetic forces in the form given by eq. (A.46), Appendix I, it is found that:

$$Q_0 = \bar{Q} \, \frac{2 n n_3 A_T B_0^2}{C} \left[1 + \tfrac{1}{2} \left(f_{c1}^2 + f_{s1}^2 + f_{c2}^2 + f_{s2}^1 \right) \right] \tag{F.65}$$

$$Q_0' = 0 \tag{F.66}$$

$$Q = -\bar{Q} \, \frac{2 n n_3 A_T B_0^2}{C} \left(2 f_{c1} + f_{c1} f_{c2} + f_{s1} f_{s2} \right) \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Bmatrix} \tag{F.67}$$

$$q = -\bar{Q} \, \frac{2 n n_3 A_T B_0^2}{C} \left(2 f_{s1} + f_{c1} f_{s2} - f_{s1} f_{c2} \right) \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Bmatrix} \tag{F.68}$$

which shows how to prepare the input for the two programs.

APPENDIX VII: Units and Dimensions

The electric and magnetic units are summarized in this section.

Magnetomotive Force

$$\mathcal{F} = Ni \quad \text{amp-turns}$$

with i = electric current, amp

    N = number of turns   (G.1)

Reluctance

$$\mathcal{R} = \frac{h}{\mu A}$$

h = length in the direction of flux, meter

A = cross-sectional area, sq. meter

μ = permeability = $4\pi \cdot 10^{-7}$ for air   (G.2)

Flux

$$\varphi = \frac{\mathcal{F}}{\mathcal{R}} \quad \text{Weber} \quad = \frac{\mathcal{F}}{\mathcal{R}} \cdot 10^{8} \quad \text{lines}$$

with $\mathcal{F}$ in   amp – turns

and $\mathcal{R}$ from eq. (G.2)   (G.3)

Induced Voltage

$$e = -N \frac{d\varphi}{dt} \quad \text{volt}$$

with φ in Weber

N is the number of turns   (G.4)

t = time, seconds

Magnetic Force

$$F = \bar{Q} \, B^2 A \quad \text{lbs}$$

with $\bar{Q} = 1/72,130,000$

B in   lines/in^2 (G.5)

A in   in^2

APPENDIX VIII: The Impedance of an Arbitrary, Elastic Rotor Supported in Flexible, Damped Bearings

In determining the stability threshold and the amplitude of a rotor with timevarying magnetic forces it is necessary to calculate the response of the rotor to high frequency excitation. In the more common case of a rotor response to mechanical unbalance it is customary to neglect the contribution from shear force, but at high frequencies this contribution becomes important and must be included. Thus, the previous analysis given in Volume 5 will be extended to include the effect of shear force and, in addition, the new analysis will take into account the actual mass distribution along the rotor.

Referring to Figure 4, let the rotor be subdivided into sections such that each shaft section has uniform diameter and uniform material properties. The endpoints of the sections are called rotor stations. Stations are introduced not only where the shaft diameter changes, but also where there are concentrated masses like wheels, impellers or sleeves, where there are bearings, where there are magnetic forces and at the endpoints of the rotor. Hence, the arbitrary rotor station n can be assigned a mass m_n, a polar and a transverse mass moment of inertia, I_{P_n} and I_{T_n}, 8 bearing coefficients: $K_{xx,n}, B_{xxn}, K_{xyn}, B_{xyn}, K_{yxn}, B_{yxn}, K_{yy_n}$ and B_{yy_n}, and magnetic forces (some or all of these quantities may be zero at any particular station). This results in an abrupt change in both the bending moment M and in the shear force V across a rotor station.

Introduce a cartesian coordinate system with the Z-axis along the rotor, the x-axis vertical downwards and the y-axis horizontal. At each point along the rotor the origin of the coordinate system coincides with the steady-state position of the rotor axis. The rotor amplitudes, caused by the applied dynamic forces, are therefore x and y. They are functions of Z. Consider rotor station n:
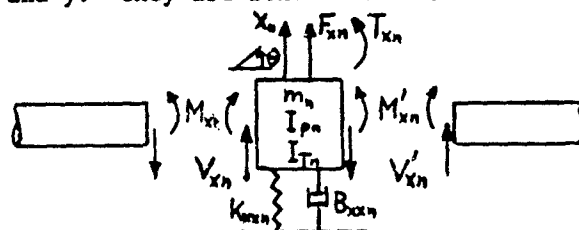
Figure 19: Force Diagram for Rotor Station n

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where Θ_n is the slope of the bent rotor (in the y-plane the slope is φ_n), F_{xn} is the x-component of the externally applied force and T_{xn} is the x-component of the externally applied moment. A force balance yields:

$$m_n \frac{d^2 x_n}{dt^2} = V_{xn} - V'_{xn} - (K_{xn} - Q_0)x_n - B_{xxn}\frac{dx_n}{dt} - K'_{xyn}y_n - B_{xyn}\frac{dy_n}{dt} + F_{xn} \qquad (H.1)$$

$$m_n \frac{d^2 y_n}{dt^2} = V_{yn} - V'_{yn} - K_{yxn}x_n - B_{yxn}\frac{dx_n}{dt} - (K_{yyn} - Q_0)y_n - B_{yyn}\frac{dy_n}{dt} + F_{yn} \qquad (H.2)$$

A moment balance yields:

$$I_{Tn}\frac{d^2\varphi_n}{dt^2} + \omega I_{Pn}\frac{d\varphi_n}{dt} = M'_{xn} - M_{xn} + Q'_0\Theta_n + T_{xn} \qquad (H.3)$$

$$I_{Tn}\frac{d^2\varphi_n}{dt^2} - \omega I_{Pn}\frac{d\Theta_n}{dt} = M'_{yn} - M_{yn} + Q'_0\varphi_n + T_{yn} \qquad (H.4)$$

where ω is the angular speed of the rotor, and Q_0 and Q'_0 are the static gradients of the magnetic force and the magnetic moment.

Assume next that the motion takes place with a given frequency ν such that:

$$X_n = X_{cn}\cos(\nu t) - X_{sn}\sin(\nu t) = Re\{(X_{cn} + iX_{sn})e^{i\nu t}\} \qquad (H.5)$$

and similarly for $y_n, \Theta_n, \varphi_n, M_{xn}, M_{yn}, V_{xn}, V_{yn}, F_{xn}, F_{yn}, T_{xn}$ and T_{yn}. Hence, eqs. (H.1) to (H.4) can be written:

$$V'_{xcn} = V_{xcn} + \left(\left(\tfrac{\nu}{\omega}\right)^2 m_n\omega^2 - K_{xxn} + Q_0\right)x_{cn} + \left(\tfrac{\nu}{\omega}\right)\omega B_{xxn}x_{sn} - K_{xyn}y_{cn} + \left(\tfrac{\nu}{\omega}\right)\omega B_{xyn}y_{sn} + F_{xcn} \qquad (H.6)$$

$$V'_{xsn} = V_{xsn} - \left(\tfrac{\nu}{\omega}\right)\omega B_{xxn}x_{cn} + \left(\left(\tfrac{\nu}{\omega}\right)^2 m_n\omega^2 - K_{xxn} + Q_0\right)x_{sn} - \left(\tfrac{\nu}{\omega}\right)\omega B_{xyn}y_{cn} - K_{xyn}y_{sn} + F_{xsn} \qquad (H.7)$$

$$V'_{ycn} = V_{ycn} - K_{yxn}x_{cn} + \left(\tfrac{\nu}{\omega}\right)\omega B_{yxn}x_{sn} + \left(\left(\tfrac{\nu}{\omega}\right)^2 m\omega^2 - K_{yyn} + Q_0\right)y_{cn} + \left(\tfrac{\nu}{\omega}\right)\omega B_{yyn}y_{sn} + F_{ycn} \qquad (H.8)$$

$$V'_{ysn} = V_{ysn} - \left(\tfrac{\nu}{\omega}\right)\omega B_{yxn}x_{cn} - K_{yxn}x_{sn} - \left(\tfrac{\nu}{\omega}\right)\omega B_{yyn}y_{cn} + \left(\left(\tfrac{\nu}{\omega}\right)^2 m\omega^2 - K_{yyn} + Q_0\right)y_{sn} + F_{ysn} \qquad (H.9)$$

$$M'_{xcn} = M_{xcn} - \left(\left(\tfrac{x}{\omega}\right)^2 I_{Tn} \omega^2 + Q'_0\right) \theta_{cn} - \left(\tfrac{x}{\omega}\right) I_{pn} \omega^2 \varphi_{sn} - T_{xcn} \tag{H.10}$$

$$M'_{xsn} = M_{xsn} - \left(\left(\tfrac{x}{\omega}\right)^2 I_{Tn} \omega^2 + Q'_0\right) \theta_{sn} + \left(\tfrac{x}{\omega}\right) I_{pn} \omega^2 \varphi_{cn} - T_{xsn} \tag{H.11}$$

$$M'_{ycn} = M_{ycn} + \left(\tfrac{x}{\omega}\right) I_{pn} \omega^2 \theta_{sn} - \left[\left(\tfrac{x}{\omega}\right)^2 I_{Tn} \omega^2 + Q'_0\right] \varphi_{cn} - T_{ycn} \tag{H.12}$$

$$M'_{ysn} = M_{ysn} - \left(\tfrac{x}{\omega}\right) I_{pn} \omega^2 \theta_{cn} - \left[\left(\tfrac{x}{\omega}\right)^2 I_{Tn} \omega^2 + Q'_0\right] \varphi_{sn} - T_{ysn} \tag{H.13}$$

This can also be written in a more compact form by expressing all the quantities in the same way as in eq. (H.5) and with the convention that only the real part applies:

$$V'_{xn} = V_{xn} + \left[\left(\tfrac{x}{\omega}\right)^2 m_n \omega^2 - K_{xxn} + Q_0 - i\left(\tfrac{x}{\omega}\right)\omega B_{xxn}\right] x_n - \left(K_{xyn} + i\left(\tfrac{x}{\omega}\right)\omega B_{xyn}\right) y_n + F_{xn} \tag{H.14}$$

$$V'_{yn} = V_{yn} - \left(K_{yxn} + i\left(\tfrac{x}{\omega}\right)\omega B_{yxn}\right) x_n + \left[\left(\tfrac{x}{\omega}\right)^2 m_n \omega^2 - K_{yyn} + Q_0 - i\left(\tfrac{x}{\omega}\right)\omega B_{yyn}\right] y_n + F_{yn} \tag{H.15}$$

$$M'_{xn} = M_{xn} - \left[\left(\tfrac{x}{\omega}\right)^2 I_{Tn} \omega^2 + Q'_0\right] \theta_n + i\left(\tfrac{x}{\omega}\right) I_{pn} \omega^2 \varphi_n - T_{xn} \tag{H.16}$$

$$M'_{yn} = M_{yn} - i\left(\tfrac{x}{\omega}\right) I_{pn} \omega^2 \theta_n - \left[\left(\tfrac{x}{\omega}\right)^2 I_{Tn} \omega^2 + Q'_0\right] \varphi_n - T_{yn} \tag{H.17}$$

Having established the change in shear force and bending moment across a rotor station(eqs.(H.6) to (H.13) or eqs. (H.14) to (H.17)), the shaft sections connecting the rotor stations will be considered. The governing equations are (ref. 1):

shaft deflection: $\dfrac{\partial x}{\partial z} = \theta - \dfrac{V_x}{\alpha A G}$ (H.19)

shaft bending: $\dfrac{\partial \theta}{\partial z} = \dfrac{1}{EI} M_x$ (P.20)

force balance $\rho A \dfrac{\partial^2 x}{\partial t^2} = -\dfrac{\partial V_x}{\partial z}$ (H.21)

moment balance $\dfrac{\partial M_x}{\partial z} = V_x$ (K.22)

where x is the amplitude, θ is the rotation angle, M is the bending moment, V is the shear force, A is the cross-sectional area, I is the transverse moment of inertia of cross-section, ρ is the mass density, E is Youngs modulus, G is the shear modulus and α is a shape factor for shear ($\alpha \cong 0.75$ for circular cross-section). Analogous equations hold for the y-direction. It should be noted that rotary inertia and gyroscopic moments have been ignored in the last equation above because these contributions are rather small and can be accounted for at the rotor stations.

These equations can be combined. Substitute for V_x from eq. (H.22) into eq. (H.19) and differentiate with respect to z:

$$\frac{\partial^2 x}{\partial z^2} = \frac{\partial \theta}{\partial z} - \frac{1}{\alpha A G}\frac{\partial^2 M_x}{\partial z^2} \tag{H.23}$$

Substitute for $\frac{\partial \theta}{\partial z}$ from eq. (H.20) and differentiate twice with respect to t:

$$\frac{\partial^4 x}{\partial z^2 \partial t^2} = \frac{1}{EI}\frac{\partial^2 M_x}{\partial t^2} - \frac{1}{\alpha A G}\frac{\partial^4 M_x}{\partial z^2 \partial t^2} \tag{H.24}$$

Next, differentiate eq. (H.22) with respect to z and substitute into eq. (H.21):

$$\rho A \frac{\partial^2 x}{\partial t^2} = -\frac{\partial^2 M_x}{\partial z^2} \tag{H.25}$$

or:

$$\frac{\partial^4 x}{\partial z^2 \partial t^2} = -\frac{1}{\rho A}\frac{\partial^4 M_x}{\partial z^4} \tag{H.26}$$

By equating the two expressions for $\frac{\partial^4 x}{\partial z^2 \partial t^2}$, the final equation becomes:

$$\frac{\partial^4 M_x}{\partial z^4} - \frac{\rho A}{\alpha A G}\frac{\partial^4 M_x}{\partial z^2 \partial t^2} + \frac{\rho A}{EI}\frac{\partial^2 M_x}{\partial t^2} = 0 \tag{H.27}$$

Let the motion be harmonic with frequency ν, i.e. $\frac{\partial^2 M_x}{\partial t^2} = -\nu^2 M_x$. Furthermore, define:

$$\beta^4 = \frac{\nu^2 \rho A}{EI} \tag{H.28}$$

$$\delta^2 = \frac{EI}{2\alpha A G} \tag{H.29}$$

Thereby eq. (H.27) can be written:

$$\frac{d^4 M_x}{dz^4} + 2\delta^2\beta^4 \frac{d^2 M_x}{dz^2} - \beta^4 M_x = 0 \qquad (H.30)$$

The characteristic equation is:

$$s^4 + 2\delta^2\beta^4 s^2 - \beta^4 = 0 \qquad (H.31)$$

with the roots:

$$s^2 = -\delta^2\beta^4 \pm \sqrt{\beta^4 + (\delta^2\beta^4)^2} = \beta^2\left[-(\delta\beta)^2 \pm \sqrt{1+(\delta\beta)^4}\right]$$

Set:

$$\left.\begin{matrix}\beta_1 \\ \\ \beta_2\end{matrix}\right\} = \beta\left[\sqrt{1+(\delta\beta)^4} \mp (\delta\beta)^2\right]^{1/2} \qquad (H.32)$$

whereby the four roots become:

$$S_1 = \beta_1 \qquad S_2 = -\beta_1 \qquad S_3 = i\beta_2 \qquad S_4 = -i\beta_2$$

and the final solution can be written:

$$\frac{1}{EI} M_x = C_1 \cosh(\beta_1 z) + C_2 \sinh(\beta_1 z) + C_3 \cos(\beta_2 z) + C_4 \sin(\beta_2 z) \qquad (H.33)$$

The three other variables become:

$$\frac{1}{EI} V_x = \frac{1}{EI} \frac{\partial M_x}{\partial z} = C_1\beta_1 \sinh(\beta_1 z) + C_2\beta_1 \cosh(\beta_1 z) - C_3\beta_2 \sin(\beta_2 z) + C_4\beta_2 \cos(\beta_2 z) \qquad (H.34)$$

$$x = \frac{1}{v^2\beta^4 A} \frac{\partial V_x}{\partial z} = \frac{1}{\beta^4 EI} \frac{\partial V_x}{\partial z} = \frac{C_1}{\beta_2^2} \cosh(\beta_1 z) + \frac{C_2}{\beta_2^2} \sinh(\beta_1 z) - \frac{C_3}{\beta_1^2} \cos(\beta_2 z) - \frac{C_4}{\beta_1^2} \sin(\beta_2 z) \qquad (H.35)$$

$$\Theta = \int \frac{M}{EI} dz = \frac{C_1}{\beta_1} \sinh(\beta_1 z) + \frac{C_2}{\beta_1} \cosh(\beta_1 z) + \frac{C_3}{\beta_2} \sin(\beta_2 z) - \frac{C_4}{\beta_2} \cos(\beta_2 z) \qquad (H.36)$$

For the shaft section of length ℓ_n between rotor stations n and (n+1), the end conditions are:

$$\underline{at\ z=0} \qquad x = x_n \qquad \Theta = \Theta_n \qquad M_x = M'_{xn} \qquad V_x = V'_{xn}$$

$$\underline{at\ z=\ell_n} \qquad x = x_{n+1} \qquad \Theta = \Theta_{n+1} \qquad M_x = M_{x,n+1} \qquad V_x = V_{x,n+1}$$

Set:

$$\lambda_1 = \ell_n \beta_1 \qquad\qquad \lambda_2 = \ell_n \beta_2 \qquad\qquad (H.37)$$

Then the four constants C_1, C_2, C_3, and C_4 are determined from the equations:

$$C_1 + C_3 = \frac{1}{EI} M'_{xn} \qquad\qquad \frac{1}{\beta_1} C_2 - \frac{1}{\beta_2} C_4 = \Theta_n$$

$$\frac{1}{\beta_2^2} C_1 - \frac{1}{\beta_1^2} C_3 = x_n \qquad\qquad \beta_1 C_2 + \beta_2 C_4 = \frac{1}{EI} V'_{xn}$$

or:

$$C_1 = \frac{\beta_2^2}{\beta_1^2 + \beta_2^2} \left[\beta_1^2 x_n + \frac{1}{EI} M'_{xn} \right] \qquad\qquad (H.38)$$

$$C_2 = \frac{\beta_1^2}{\beta_1^2 + \beta_2^2} \left[-\beta_2^2 x_n + \frac{1}{EI} M'_{xn} \right] \qquad\qquad (H.39)$$

$$C_3 = \frac{\beta_1}{\beta_1^2 + \beta_2^2} \left[\beta_2^2 \Theta_n + \frac{1}{EI} V'_{xn} \right] \qquad\qquad (H.40)$$

$$C_4 = \frac{\beta_2}{\beta_1^2 + \beta_2^2} \left[-\beta_1^2 \Theta_n + \frac{1}{EI} V'_{xn} \right] \qquad\qquad (H.41)$$

Substituting for C_1 to C_4 into eqs. (H.33) to (H.36) and setting $z=\ell_n$ yields:

$$x_{n+1} = \frac{1}{\beta_1^2 + \beta_2^2} \left\{ \left[\beta_1^2 \cosh\lambda_1 + \beta_2^2 \cos\lambda_2 \right] x_n + \left[\beta_1 \sinh\lambda_1 + \beta_2 \sin\lambda_2 \right] \Theta_n + \left[\cosh\lambda_1 - \cos\lambda_2 \right] \frac{1}{EI} M'_{xn} \right. \qquad (H.42)$$

$$\left. + \left[\frac{\beta_1}{\beta_2^2} \sinh\lambda_1 - \frac{\beta_2}{\beta_1^2} \sin\lambda_2 \right] \frac{1}{EI} V'_{xn} \right\}$$

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$$\theta_{n+1} = \frac{1}{\beta_1^2+\beta_2^2}\left\{\left[\beta_1\beta_2^2\sinh\lambda_1-\beta_1^2\beta_2\sin\lambda_2\right]x_n+\left[\beta_2^2\cosh\lambda_1+\beta_1^2\cos\lambda_2\right]\theta_n+\left[\frac{\beta_2^2}{\beta_1}\sinh\lambda_1+\frac{\beta_1^2}{\beta_2}\sin\lambda_2\right]\frac{1}{EI}M'_{xn}\right.$$
$$\left.+\left[\cosh\lambda_1-\cos\lambda_2\right]\frac{1}{EI}V'_{xn}\right\} \qquad (H.43)$$

$$\frac{1}{EI}M_{x,n+1} = \frac{1}{\beta_1^2+\beta_2^2}\left\{\beta_1^2\beta_2^2\left[\cosh\lambda_1-\cos\lambda_2\right]x_n+\beta_1\beta_2\left[\beta_2\sinh\lambda_1-\beta_1\sin\lambda_2\right]\theta_n+\left[\beta_2^2\cosh\lambda_1+\beta_1^2\cos\lambda_2\right]\frac{1}{EI}M'_{xn}\right.$$
$$\left.+\left[\beta_1\sinh\lambda_1+\beta_2\sin\lambda_2\right]\frac{1}{EI}V'_{xn}\right\} \qquad (H.44)$$

$$\frac{1}{EI}V_{x,n+1} = \frac{1}{\beta_1^2+\beta_2^2}\left\{\beta_1^2\beta_2^2\left[\beta_1\sinh\lambda_1+\beta_2\sin\lambda_2\right]x_n+\beta_1^2\beta_2^2\left[\cosh\lambda_1-\cos\lambda_2\right]\theta_n+\beta_1\beta_2\left[\beta_2\sinh\lambda_1-\beta_1\sin\lambda_2\right]\frac{1}{EI}M'_{xn}\right.$$
$$\left.+\left[\beta_1^2\cosh\lambda_1+\beta_2^2\cos\lambda_2\right]\frac{1}{EI}V'_{xn}\right\} \qquad (H.45)$$

These equations can be written:

$$X_{n+1} = a_{1n}X_n + a_{3n}\theta_n + a_{4n}M'_{xn} + a_{7n}V'_{xn}$$
$$\theta_{n+1} = a_{5n}X_n + a_{2n}\theta_n + a_{6n}M'_{xn} + a_{4n}V'_{xn}$$
$$M_{x,n+1} = a_{9n}X_n + a_{10n}\theta_n + a_{2n}M'_{xn} + a_{3n}V'_{xn} \qquad (H.46)$$
$$V_{x,n+1} = a_{8n}X_n + a_{9n}\theta_n + a_{5n}M'_{xn} + a_{1n}V'_{xn}$$

Because:

$$\beta_1\beta_2 = \beta^2$$

the 10 coefficients a_{1n} to a_{10n} become:

$$a_{1n} = \frac{1}{\beta_1^2+\beta_2^2}\left[\beta_1^2\cosh\lambda_1+\beta_2^2\cos\lambda_2\right] \qquad (H.47)$$

$$a_{2n} = \frac{1}{\beta_1^2+\beta_2^2}\left[\beta_2^2\cosh\lambda_1+\beta_1^2\cos\lambda_2\right] \qquad (H.48)$$

$$a_{3n} = \frac{1}{\beta_1^2+\beta_2^2}\left[\beta_1\sinh\lambda_1+\beta_2\sin\lambda_2\right] \qquad (H.49)$$

$$a_{4n} = \frac{1}{\beta_1^2+\beta_2^2}\left[\cosh\lambda_1-\cos\lambda_2\right]\frac{1}{EI} \qquad (H.50)$$

$$a_{5n} = \frac{\beta^2}{\beta_1^2+\beta_2^2}\left[\beta_2\sinh\lambda_1-\beta_1\sin\lambda_2\right] \qquad (H.51)$$

$$a_{6n} = \frac{1}{\beta^2(\beta_1^2+\beta_2^2)}\left[\beta_2^3\sinh\lambda_1+\beta_1^3\sin\lambda_2\right]\frac{1}{EI} \qquad (H.52)$$

$$a_{7n} = \frac{1}{\beta^2(\beta_1^2+\beta_{2;}^{2})}\left[\beta_1^3 \sinh\lambda_1 - \beta_2^3 \sin\lambda_2\right]\frac{1}{EI} \qquad (\text{H.}53)$$

$$a_{8n} = \nu^2 \rho A\, a_{3n} \qquad (\text{H.}54)$$

$$a_{9n} = \nu^2 \rho A\, EI\, a_{4n} \qquad (\text{H.}55)$$

$$a_{10n} = EI\, a_{5n} \qquad (\text{H.}56)$$

These 10 coefficients are different for each shaft section since A, I and ℓ_n vary between sections.

In the limit, as λ_1 and λ_2 become very small, the following relationships hold:

$$\begin{aligned}
\cosh\lambda_1 &\longrightarrow 1 + \tfrac{1}{2}\beta_1^2 \ell_n^2 + \tfrac{1}{24}\beta_1^4 \ell_n^4 \\
\cos\lambda_2 &\longrightarrow 1 - \tfrac{1}{2}\beta_2^2 \ell_n^2 + \tfrac{1}{24}\beta_2^4 \ell_n^4 \\
\sinh\lambda_1 &\longrightarrow \beta_1 \ell_n\left[1 + \tfrac{1}{6}\beta_1^2 \ell_n^2 + \tfrac{1}{120}\beta_1^4 \ell_n^4\right] \\
\sin\lambda_2 &\longrightarrow \beta_2 \ell_n\left[1 - \tfrac{1}{6}\beta_2^2 \ell_n^2 + \tfrac{1}{120}\beta_2^4 \ell_n^4\right]
\end{aligned} \qquad (\text{H.}57)$$

Hence, in the limit:

$$\begin{aligned}
a_{1n} &\longrightarrow 1 & a_{2n} &\longrightarrow 1 & a_{3n} &\longrightarrow \ell_n \\
a_{4n} &\longrightarrow \frac{\ell_n^2}{2EI} & a_{5n} &\longrightarrow \tfrac{1}{6}\beta^4 \ell_n^3 = \nu^2 \rho A \frac{\ell_n^3}{6EI} = \tfrac{1}{6}\frac{\beta^4 \ell_n^4}{\ell_n} \longrightarrow 0 \\
a_{6n} &\longrightarrow \frac{\ell_n}{EI} & a_{7n} &\longrightarrow \frac{\ell_n^3}{6EI} - \frac{\ell_n}{\alpha AG} \\
a_{8n} &\longrightarrow \nu^2 \rho A\, \ell_n & a_{9n} &\longrightarrow \tfrac{1}{2}\nu^2 \rho A\, \ell_n^2 & a_{10n} &\longrightarrow \tfrac{1}{6}\nu^2 \rho A\, \ell_n^3
\end{aligned} \qquad (\text{H.}58)$$

From eq. (H.57) it is seen that these limits are exact when:

$$\tfrac{1}{24}\beta^4 \ell_n^4 < 10^{-8} \qquad \text{or} \qquad \beta\ell \lesssim 0.022 \qquad (\text{H.}59)$$

assuming that the computer works with 8 significant figures. Under these limit conditions and if ℓ_n is not too small, eqs. (H.46) reduce to:

$$\begin{aligned}
X_{n+1} &= X_n + \ell_n \theta_n + \frac{\ell_n^2}{2EI} M'_{xn} + \left[\frac{\ell_n^3}{6EI} - \frac{\ell_n}{\alpha AG}\right] V'_{xn} \\
\theta_{n+1} &= \theta_n + \frac{\ell_n}{EI} M'_{xn} + \frac{\ell_n^2}{2EI} V'_{xn} \\
M_{x,n+1} &= \tfrac{1}{2}\nu^2 \rho A\, \ell_n^2 X_n + \tfrac{1}{6}\nu^2 \rho A\, \ell_n^3 \theta_n + M'_{xn} + \ell_n V'_{xn} \\
V_{x,n+1} &= \nu^2 \rho A\, \ell_n X_n + \tfrac{1}{2}\nu^2 \rho A\, \ell_n^2 \theta_n + V'_{xn}
\end{aligned} \qquad (\text{H.}60)$$

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which are the same expres ... ns as would be obtained if the shaft was considered massless and the actu ... shaft mass lumped at the endpoints as done in Volume 5.

It is seen that there is no coupling between the x-direction and the y-direction, nor between the cosine and sine-components in eqs. (H.46). Hence, eqs. (H.46 are valid also if the variables are subscripted with c or s, or for the analogous y-components. In this way, eqs. (H.14) to (H.17) together with eqs. (H.46) establish recurrence relationships by which the amplitude, etc. can be calculated step by step, starting from one end of the rotor. Let the rotor end at Station 1 be free, i.e. $M_{x_1} = M_{y_1} = V_{x_1} = V_{y_1} = 0$. First, set $x_1 = 1$ and $y_1 = \Theta_1 = \varphi_1 = 0$, and use the recurrence formulas to calculate the bending moment and shear force at the last station, station m. Denote the values as: $M'_{x_m} = a_{11}$, $M'_{y_m} = a_{21}$, $V'_{x_m} = a_{31}$, $V'_{y_m} = a_{41}$ (the a's are complex). Next, set $y_1 = 1$ and $x_1 = \Theta_1 = \varphi_1 = 0$, and calculate $M'_{x_m} = a_{12}$, $M'_{y_m} = a_{22}$, $V'_{x_m} = a_{32}$, $V'_{y_m} = a_{42}$. Repeat the calculations with $\Theta_1 = 1$ and $\varphi_1 = 1$, respectively. Finally, perform four additional calculations with $x_1 = y_1 = \Theta_1 = \varphi_1 = 0$, the first with $F_x = 1$, $F_y = T_x = T_y = 0$, the second with $F_y = 1$, $F_x = T_x = T_y = 0$, and so on where the forces and moments are applied at that rotor station where the magnetic forces act. Assuming the rotor end at station m to be free, we have $M'_{x_m} = M'_{y_m} = V'_{x_m} = V'_{y_m} = 0$, i.e.:

$$\begin{Bmatrix} M'_{x_m} \\ M'_{y_m} \\ V'_{x_m} \\ V'_{y_m} \end{Bmatrix} = \begin{Bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{Bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \Theta_1 \\ \varphi_1 \end{Bmatrix} + \begin{Bmatrix} a_{15} & a_{16} & a_{17} & a_{18} \\ a_{25} & a_{26} & a_{27} & a_{28} \\ a_{35} & a_{36} & a_{37} & a_{38} \\ a_{45} & a_{46} & a_{47} & a_{48} \end{Bmatrix} \begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = 0 \quad \text{(H.61)}$$

or:

$$A X_1 = B F \qquad \text{(H.62)}$$

where:

$$X_1 = \begin{Bmatrix} x_1 \\ y_1 \\ \Theta_1 \\ \varphi_1 \end{Bmatrix} \qquad\qquad F = \begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} \qquad\qquad \text{(H.63)}$$

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and A and B are matrices defined through the above equations. They are complex such that:

$$A = A_c + i A_s$$

$$B = B_c + i B_s$$

(H.64)

and similarly for X_1 and F. To solve eq. (H.62) for X_1, write it out into its real and imaginary parts:

$$A_c X_{1c} - A_s X_{1s} = (BF)_c$$

$$A_s X_{1c} + A_c X_{1s} = (BF)_s$$

(H.65)

Solve the equations to get:

$$X_{1c} = \left[A_s^{-1} A_c + A_c^{-1} A_s \right]^{-1} \left[A_s^{-1}(B \cdot F)_c + A_c^{-1}(B \cdot F)_s \right]$$

$$X_{1s} = \left[A_s^{-1} A_c + A_c^{-1} A_s \right]^{-1} \left[-A_c^{-1}(BF)_c + A_s^{-1}(B \cdot F)_s \right]$$

(H.66)

Define:

$$A^{-1} = \left[A_s^{-1} A_c + A_c^{-1} A_s \right]^{-1} \left[A_s^{-1} - i A_c^{-1} \right]$$

(H.67)

whereby the solution of eq. (H.62) becomes:

$$X_1 = A^{-1} B F$$

(H.68)

Let the magnetic forces act at a station with amplitudes:

$$X = \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix}$$

(H.69)

In performing the rotor calculations described above, X is expressed by a matrix equation similar to eq. (H.61):

$$X = C X_1 + D F$$

(H.70)

Substituting for X, from eq. (H.68):

$$X = \left[C A^{-1} B + D \right] F = E^{-1} F \qquad (H.71)$$

where $E^{-1} = \left[C A^{-1} B + D \right]$ is a complex matrix with 4 rows and 4 columns.

Solve this equation to get:

$$F = \begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = E X = \begin{Bmatrix} (\varkappa_{11} + i\lambda_{11}) - - - - (\varkappa_{14} + i\lambda_{14}) \\ \vdots \qquad\qquad \vdots \\ \vdots \qquad\qquad \vdots \\ (\varkappa_{41} + i\lambda_{41}) - - - - (\varkappa_{44} + i\lambda_{44}) \end{Bmatrix} \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix} \qquad (H.72)$$

where E is obtained as the inverse of E^{-1} by the same method used to invert A (see eq. (H.67)). Here:

$$(\varkappa_{11} + i\lambda_{11}) x = (\varkappa_{11} + i\lambda_{11})(x_c + i x_s) = (\varkappa_{11} x_c - \lambda_{11} x_s) \cos(\nu t) - (\lambda_{11} x_c + \varkappa_{11} x_s) \sin(\nu t) \quad (H.73)$$

and so on. The E- matrix expresses the impedance of the rotor for the chosen frequency ν at that rotor station where the magnetic forces and moments are applied. It is used in the rotor stability calculation and the rotor response calculation as discussed in Appendices IX and X.

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APPENDIX IX: Calculation of the Threshold of Instability for a Rotor with Magnetic Forces

In Appendix VIII it is shown that the rotor can be represented by an impedance matrix E at the point where the magnetic forces and moments are applied. This matrix depends on the vibratory frequency ν and relates the rotor amplitudes and slopes to the imposed forces (see eq. (H.72), Appendix VIII) The magnetic forces F_x and F_y and moments, T_x and T_y on the other hand, also depend on the rotor amplitudes x and y and slopes θ and φ, and can be written:

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = -\left[\begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{x\theta} & Q_{x\varphi} \\ Q_{yx} & Q_{yy} & Q_{y\theta} & Q_{y\varphi} \\ Q_{\theta x} & Q_{\theta y} & Q_{\theta\theta} & Q_{\theta\varphi} \\ Q_{\varphi x} & Q_{\varphi y} & Q_{\varphi\theta} & Q_{\varphi\varphi} \end{Bmatrix} \cos(\Omega t) - \begin{Bmatrix} q_{xx} & q_{xy} & q_{x\theta} & q_{x\varphi} \\ q_{yx} & q_{yy} & q_{y\theta} & q_{y\varphi} \\ q_{\theta x} & q_{\theta y} & q_{\theta\theta} & q_{\theta\varphi} \\ q_{\varphi x} & q_{\varphi y} & q_{\varphi\theta} & q_{\varphi\varphi} \end{Bmatrix} \sin(\Omega t) \right] \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix} \qquad (J.1)$$

where Ω radians/sec is the frequency of the magnetic forces, and the Q's and q's are the gradients of the magnetic forces and moments. In most cases, several of the gradients are zero and there also exist certain symmetry relationships between the gradients. However, all the terms will be kept in the analysis to make it general.

Combining eqs. (J.1) and eq. (H.72), Appendix VIII, a matrix equation is obtained with the rotor amplitudes x and y and the rotor slopes θ and φ as the unknowns.

To solve the equation it is necessary to expand x, y, θ and φ in Fourier series as:

$$x = \sum_{k=0}^{\infty} \left[x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi) \right] \qquad (J.2)$$

$$y = \sum_{k=0}^{\infty} \left[y_{ck} \cos(k\psi) - y_{sk} \sin(k\psi) \right] \qquad (J.3)$$

and similarly for θ and φ where:

$$\psi = \tfrac{1}{2} \Omega t \qquad (J.4)$$

Thus, for a given k the frequency is:

$$\nu = k\,\frac{\Omega}{2} \qquad\qquad (J.5)$$

and the elements of the impedance matrix E (i.e. the χ's and λ's of eq. (H.72), Appendix VIII)are evaluated at these frequencies such that there will be and E-matrix for each value of k:

$$E_k = E_{C_k} + i\,E_{S_k} \qquad\qquad (J.6)$$

Next, define:

$$X_k = \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix}_k = X_{C_k} + i\,X_{S_k} \qquad\qquad (J.7)$$

where:

$$X_{C_k} = \begin{Bmatrix} x_{c_k} \\ y_{c_k} \\ \theta_{c_k} \\ \varphi_{c_k} \end{Bmatrix} \qquad\qquad X_{S_k} = \begin{Bmatrix} x_{s_k} \\ y_{s_k} \\ \theta_{s_k} \\ \varphi_{s_k} \end{Bmatrix} \qquad\qquad (J.8)$$

Also, set:

$$Q = \begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{x\theta} & Q_{x\varphi} \\ Q_{yx} & Q_{yy} & Q_{y\theta} & Q_{y\varphi} \\ Q_{\theta x} & Q_{\theta y} & Q_{\theta\theta} & Q_{\theta\varphi} \\ Q_{\varphi x} & Q_{\varphi y} & Q_{\varphi\theta} & Q_{\varphi\varphi} \end{Bmatrix} \qquad\qquad (J.9)$$

$$q = \begin{Bmatrix} q_{xx} & q_{xy} & q_{x\theta} & q_{x\varphi} \\ q_{yx} & q_{yy} & q_{y\theta} & q_{y\varphi} \\ q_{\theta x} & q_{\theta y} & q_{\theta\theta} & q_{\theta\varphi} \\ q_{\varphi x} & q_{\varphi y} & q_{\varphi\theta} & q_{\varphi\varphi} \end{Bmatrix} \qquad\qquad (J.10)$$

With these definitions, eqs. (H.72) and (J.1) can be combined to yield:

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \sum_{k=0}^{\infty} E_k X_k \big[\cos(k\psi) + i\sin(k\psi)\big] = -\big[Q\cos(2\psi) - q\sin(2\psi)\big] \sum_{k=0}^{\infty} X_k \big[\cos(k\psi) - \sin(k\psi)\big] \quad (J.11)$$

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or in expanded form:

$$\sum_{k=0}^{\infty} \left[\left(E_{Ck} X_{Ck} - E_{Sk} X_{Sk} \right) \cos(k\psi) - \left(E_{Sk} X_{Ck} + E_{Ck} X_{Sk} \right) \sin(k\psi) \right]$$

$$+ \tfrac{1}{2} Q \sum_{k=0}^{\infty} X_{Ck} \left[\cos(k+2)\psi + \cos(k-2)\psi \right] - \tfrac{1}{2} q \sum_{k=0}^{\infty} X_{Sk} \left[\cos(k+2)\psi - \cos(k-2)\psi \right]$$

$$- \tfrac{1}{2} q \sum_{k=0}^{\infty} X_{Ck} \left[\sin(k+2)\psi - \sin(k-2)\psi \right] - \tfrac{1}{2} Q \sum_{k=0}^{\infty} X_{Sk} \left[\sin(k+2)\psi + \sin(k-2)\psi \right] = 0 \qquad (J.12)$$

By collecting terms in $\cos(k\psi)$ and $\sin(k\psi)$, two sets of equations are obtained, one set for k even ($k=0,2,4, - - -$) and one set for k odd ($k=1,3,5, - - -$).

Consider first the case of k even. When $k \geq 4$, eq. (J.12) yields for any k:

k even, $k \geq 4$

$$E_{Ck} X_{Ck} - E_{Sk} X_{Sk} + \tfrac{1}{2} Q X_{C,k-2} - \tfrac{1}{2} q X_{S,k-2} + \tfrac{1}{2} Q X_{C,k+2} + \tfrac{1}{2} q X_{S,k+2} = 0 \qquad (J.13)$$

$$E_{Sk} X_{Ck} + E_{Ck} X_{Sk} + \tfrac{1}{2} q X_{C,k-2} + \tfrac{1}{2} Q X_{S,k-2} - \tfrac{1}{2} q X_{C,k+2} + \tfrac{1}{2} Q X_{S,k+2} = 0$$

which can be written:

$$\left(E_{Ck} + i E_{Sk} \right) \left(X_{Ck} + i X_{Sk} \right) + \tfrac{1}{2} (Q+iq)(X_{C,k-2} + i X_{S,k-2}) + \tfrac{1}{2}(Q-iq)(X_{C,k+2} + i X_{S,k+2}) = 0 \quad (J.14)$$

or:

$$G X_{k-2} + E_k X_k + H X_{k+2} = 0 \qquad (J.15)$$

where:

$$G = \tfrac{1}{2}(Q+iq) \qquad\qquad H = \tfrac{1}{2}(Q-iq) \qquad (J.16)$$

Define the matrix S_k by:

$$X_k = S_{k-2} X_{k-2} \qquad (J.17)$$

Substitute into eq. (J.15) to get:

$$G X_{k-2} + \left[E_k + H S_k \right] X_k = 0 \qquad (J.18)$$

or:

$$X_k = - \left[E_k + H S_k \right]^{-1} G X_{k-2} \qquad \underline{k \geq 4} \qquad (J.19)$$

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By comparing eqs. (J.19) and (J.17):

$$S_{k-2} = -[E_k + HS_k]^{-1} G \qquad k \geq 4 \qquad (J.20)$$

For k=2, eq. (J.12) yields:

$$2G X_{c_0} + E_2 X_2 + H X_4 = 0 \qquad (J.21)$$

since $X_{s_0} = 0$. Hence:

$$X_2 = -[E_2 + HS_2]^{-1} 2G X_{c_0} \qquad (J.22)$$

Thus, eq. (J.17) is valid also for k=2 if it is defined that:

$$S_0 = -[E_2 + HS_2]^{-1} 2G \qquad (J.23)$$

Turning last to the case of k=0, eq. (J.12) yields:

$$E_{c_0} X_{c_0} + \tfrac{1}{2} Q X_{c_2} + \tfrac{1}{2} q X_{s_2} = 0 \qquad (J.24)$$

or by introducing eq. (J.22):

$$\left[E_{c_0} + \tfrac{1}{2} Q S_{c_0} + \tfrac{1}{2} q S_{s_0} \right] X_{c_0} = 0 \qquad (J.25)$$

where:

$$S_{c_0} + i S_{s_0} = S_0 \qquad (J.26)$$

The coefficient matrix on the left hand side of eq. (J.25) is a 4 by 4 real matrix. In order for a non-trivial solution of X_{c_0} to exist it is necessary that the determinant is zero, and in that case the rotor is unstable.

Turning next to the case of k odd and taking $k \geq 3$, eq. (J.12) yields for a given value of k, equations identical to eq. (J.15). Hence eqs. (J.17) to (J.20) are

also valid for $k \geq 3$. However, for $k=1$ eq. (J.12) gives:

$$\left(E_{c_1} + \tfrac{1}{2}Q\right)X_{c_1} - \left(E_{s_1} - \tfrac{1}{2}q\right)X_{s_1} + \tfrac{1}{2}QX_{c_3} + \tfrac{1}{2}qX_{s_3} = 0$$

(J.27)

$$\left(E_{s_1} + \tfrac{1}{2}q\right)X_{c_1} + \left(E_{c_1} - \tfrac{1}{2}Q\right)X_{c_1} - \tfrac{1}{2}qX_{c_3} + \tfrac{1}{2}qX_{s_3} = 0$$

By substitution from eq. (J.17), eq. (J.27) becomes:

$$\left[E_1 + HS_1\right]X_1 + \tfrac{1}{2}QX_{c_1} + \tfrac{1}{2}qX_{s_1} + i\left(\tfrac{1}{2}qX_{c_1} - \tfrac{1}{2}QX_{s_1}\right) = 0$$

(J.28)

Equating real and imaginary parts, this equation yields an 8 by 8 real matrix whose determinant must vanish at the threshold of instability of the rotor.

In order to evaluate the two determinants, the one for even values of k from eq. (J.25) and the one for odd values of k from eq. (J.28), it is necessary to calculate S_o and S_1. This is done by using the recurrence relationship of eq. (J.20) where S_{k-2} can be found when S_k is known (for k=2, use eq. (J.23)). Now, the elements of the impedance matrix E_k are of the order k^2 (except if $k\frac{\Omega}{2}$ is close to a resonant frequency of the system). Thus, for sufficiently high values of k, S_k will be of the order k^{-2} such that it is possible to ignore all S_k-matrices for $k \geq p$ where p is selected on the basis of the desired accuracy of the calculations. With $S_p = S_{p+1} = 0$, eq. (J.20) yields:

$$S_{p-1} = -E_{p+1}^{-1}\, G$$

(J.29)

$$S_{p-2} = -E_p^{-1}\, G$$

after which eq. (J.20) and eq. (J.23) can be employed to calculate S_k, $p-3 \geq k \geq 0$, keeping the S-matrices for even values of k separate from the S-matrices for odd values of k. Once S_o and S_1 have been obtained the two determinants can be calculated.

To perform a complete stability analysis of a rotor, the rotor dimensions, the rotor speed and the bearing coefficients must be specified. It is then possible to calculate the impedance matrices of the rotor (the E_k- matrices), by the method explained in Appendix VIII Next, to determine if the rotor is stable or unstable,

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assume the magnetic force gradients (i.e. the Q's and q's of eq. (J.1)) to be variable but such that their mutual ratio is kept constant and equal to their specified value. In other words, introduce a reference value, Q_{ref} , (for instance, $Q_{ref} = Q_{xx}$) and let Q_{ref} be the single variable but such that when Q_{ref} varies, the ratios Q_{xy}/Q_{ref} , q_{xx}/Q_{ref} , and so on remain fixed. Then increase Q_{ref} in steps, starting with $Q_{ref} = 0$, and calculate the corresponding values of the two determinants as discussed above. In this way the determinants are obtained as functions of Q_{ref} . If neither of the determinants become zero between $Q_{ref} = 0$ and that value of Q_{ref} where the Q's and q's assume their specified values, the rotor is stable, otherwise unstable. It should be noted that this assumes the rotor to be stable at $Q_{ref} = 0$, i.e. when there are no magnetic forces. Even if the two determinants are not zero for $Q_{ref} = 0$, the rotor may still be unstable with hydrodynamic whirl instability induced by the fluid film forces in the bearings. This latter form of instability cannot be analyzed by the present method but must be checked by the methods given in Volume 5.

APPENDIX X: Calculation of the Amplitude Response of a Rotor with Magnetic Forces

When there is a built-in eccentricity between the rotor center and the magnetic axis of the generator stator, the magnetic forces will force the rotor to whirl. Let this built-in eccentricity be described by $(x_0, y_0, \theta_0, \varphi_0)$ where x_0 and y_0 give the coordinates of the rotor center with respect to the stator center and θ_0 and φ_0 give the angles between the rotor axis and the stator axis. These values include the contributions from the static components of the magnetic forces. Then the magnetic forces F_x and F_y and the magnetic moments T_x and T_y can be expressed in terms of the rotor amplitudes x and y, the rotor slopes θ and φ, and x_0, y_0, θ_0 and φ_0 :

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = - \left[Q \cos(\Omega t) - q \sin(\Omega t) \right] \begin{Bmatrix} x_0 + x \\ y_0 + y \\ \theta_0 + \theta \\ \varphi_0 + \varphi \end{Bmatrix} \qquad (K.1)$$

where Q and q are matrices defined by eqs. (J.9) and (J.10), Appendix IX. These forces and moments act on the rotor where they produce the amplitudes x and y and the slopes θ and φ which are related to the forces through the impedance matrix E as given by eq. (H.72), Appendix VIII. Thus, by combining eq. (H.72) and eq. (K.1) a matrix equation is obtained with x, y, θ and φ as the unknowns. To solve this equation, expand x, y, θ and φ in Fourier series:

$$x = \sum_{k=0}^{\infty} \left[x_{c_k} \cos(k\psi) - x_{s_k} \sin(k\psi) \right] \qquad (K.2)$$

and similarly for y, θ and φ where:

$$\psi = \Omega t \qquad (K.3)$$

Thus, for a given value of k the frequency is:

$$\nu = k\Omega \qquad (K.4)$$

It should be noted that this differs from the stability analyses where $\psi = \tfrac{1}{2}\Omega t$ and $\nu = k\frac{\Omega}{2}$. In the response calculation only multiples of the magnetic force frequency are considered or, in terms of the stability analysis, only the case of

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"k even" is considered.

For each k-value a rotor impedance matrix E_k can be computed as shown in Appendix VIII

$$E_k = E_{ck} + i E_{sk} \qquad (K.5)$$

Define:

$$X_k = \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix}_k = X_{ck} + i X_{sk} \qquad (K.6)$$

where:

$$X_{ck} = \begin{Bmatrix} x_{ck} \\ y_{ck} \\ \theta_{ck} \\ \varphi_{ck} \end{Bmatrix} \qquad X_{sk} = \begin{Bmatrix} x_{sk} \\ y_{sk} \\ \theta_{sk} \\ \varphi_{sk} \end{Bmatrix} \qquad (K.7)$$

Then combine eqs. (H.72) and (K.1) to get:

$$\sum_{k=0}^{\infty} E_k X_k \left[\cos(k\psi) + i\sin(k\psi) \right] = -\left[Q\cos(\Omega t) - q\sin(\Omega t) \right]\left[\begin{Bmatrix} x_o \\ y_o \\ \theta_o \\ \varphi_o \end{Bmatrix} + \sum_{k=0}^{\infty} X_k \left[\cos(k\psi) + i\sin(k\psi) \right] \right] \quad (K.8)$$

which can be expanded into the form:

$$\sum_{k=0}^{\infty} \left[(E_{ck} X_{ck} - E_{sk} X_{sk})\cos(k\psi) - (E_{sk} X_{ck} + E_{ck} X_{sk})\sin(k\psi) \right]$$

$$+ \tfrac{1}{2} Q \sum_{k=0}^{\infty} X_{ck} \left[\cos(k+1)\psi + \cos(k-1)\psi \right] - \tfrac{1}{2} q \sum_{k=0}^{\infty} X_{sk} \left[\cos(k+1)\psi - \cos(k-1)\psi \right]$$

$$- \tfrac{1}{2} q \sum_{k=0}^{\infty} X_{ck} \left[\sin(k+1)\psi - \sin(k-1)\psi \right] - \tfrac{1}{2} Q \sum_{k=0}^{\infty} X_{sk} \left[\sin(k+1)\psi + \sin(k-1)\psi \right] \qquad (K.9)$$

$$= -\left(Q\cos\psi - q\sin\psi \right) \begin{Bmatrix} x_o \\ y_o \\ \theta_o \\ \varphi_o \end{Bmatrix}$$

Collecting terms in cos($k\psi$) and sine($k\psi$), this equation gives rise to an infinite number of simultaneous equations. For $k \geq 2$ and for any arbitrary value of k, the equations become:

$\underline{k \geq 2}$

$$E_{ck} X_{ck} - E_{sk} X_{sk} + \tfrac{1}{2} Q X_{c,k-1} - \tfrac{1}{2} q X_{s,k-1} + \tfrac{1}{2} Q X_{c,k+1} + \tfrac{1}{2} q X_{s,k+1} = 0$$

$$\qquad (K.10)$$

$$E_{sk} X_{ck} + E_{ck} X_{sk} + \tfrac{1}{2} q X_{c,k-1} + \tfrac{1}{2} Q X_{s,k-1} - \tfrac{1}{2} q X_{c,k+1} + \tfrac{1}{2} Q X_{s,k+1} = 0$$

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or in complex notation:

$$\underline{k \geq 2} \qquad G X_{k-1} + E_k X_k + H X_{k+1} = 0 \qquad\qquad (K.11)$$

where the G and H matrices are defined by eq. (J.16), Appendix IX. For k=1, the right hand side is not zero. The equations become:

$$\underline{k=1}$$

$$E_{C1} X_{C1} - E_{S1} X_{S1} + Q X_{C0} + \tfrac{1}{2} Q X_{C2} + \tfrac{1}{2} q X_{S2} = -Q \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix} \qquad (K.1$$

$$E_{S1} X_{C1} + E_{C1} X_{S1} + q X_{C0} - \tfrac{1}{2} q X_{C2} + \tfrac{1}{2} Q X_{S2} = -q \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix}$$

or in complex notation:

$$2 G X_{C0} + E_1 X_1 + H X_2 = -2G \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix} \qquad\qquad (K.1$$

since $X_{S0} = 0$.

Finally, for k=0 eq. (K.9) yields:

$$E_{C0} X_{C0} + \tfrac{1}{2} Q X_{C1} + \tfrac{1}{2} q X_{S1} = 0 \qquad\qquad (K.14$$

Now, define a matrix S_k by:

$$\underline{k \geq 2} \qquad\qquad X_k = S_{k-1} X_{k-1} \qquad\qquad (K.15$$

and substitute into eq. (K.11) to get:

$$G X_{k-1} + (E_k + H S_k) X_k = 0$$

or:

$$X_k = - [E_k + H S_k]^{-1} G X_{k-1} \qquad\qquad (K.16)$$

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Hence:

$$S_{k-1} = -[E_k + HS_k]^{-1} G \qquad k \geq 2 \qquad (K.17)$$

As previously discussed, E_k is of the order k^2 which means that for sufficiently large values of k, S_k is of the order k^{-2}. Hence, for $k \geq p$ where the choice of p depends on the desired accuracy of the calculation, S_k may be set equal to zero, i.e.:

$$S_{p-1} = -E_p^{-1} G \qquad (K.18)$$

Thereafter eq. (K.17) can be used to calculate all subsequent S_k matrices, $p-2 \geq k \geq 1$. Having obtained S_1, eq. (K.12) becomes:

$$2G X_{co} + [E_1 + HS_1] X_1 = -2G \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix} \qquad (K.19)$$

with the solution:

$$X_1 = -[E_1 + HS_1]^{-1} 2G [X_{co} + \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix}] = 2S_0 [X_{co} + \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix}] \qquad (K.20)$$

where:

$$S_0 = S_{co} + i S_{so} = -[E_1 + HS_1]^{-1} G \qquad (K.21)$$

With this result, eq. (K.14) can be written:

$$[E_{co} + Q S_{co} + q S_{so}] X_{co} = -[Q S_{co} + q S_{so}] \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix} \qquad (K.22)$$

or:

$$[E_{co} + Q S_{co} + q S_{so}][X_{co} + \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix}] = E_{co} \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \varphi_0 \end{Bmatrix} \qquad (K.23)$$

These four equations may be solved for X_{c_0} or $\left[X_{c_0} + \left\{\begin{matrix} x_0 \\ \varphi_0 \\ \vdots \\ \varphi_t \end{matrix}\right\}\right]$ after which X_1 can be calculated from eq. (K.20) and all other X_k-vectors from eq. (K.15). Thereby the complete amplitude response is obtained at the station where the magnetic forces are applied. To determine the response at other stations, the rotor calculation has resulted in relationships:

at station n
$$X_{nk} = C_{nk} X_{1k} + D_{nk} F_k \qquad \text{(K.24)}$$

Here X_{1k} is related to F_k by eq. (H.63):

$$X_{1k} = A_k^{-1} B_k F_k \qquad \text{(K.25)}$$

i.e.:

$$X_{nk} = \left[C_{nk} A_k^{-1} B_k + D_{nk}\right] F_k \qquad \text{(K.26)}$$

or with F_k given by eq. (H.72):

$$X_{nk} = \left[C_{nk} A_k^{-1} B_k + D_{nk}\right] E_k X_k \qquad \text{(K.27)}$$

Thus, with X_k determined, the amplitudes and slopes at any other rotor station can be found.

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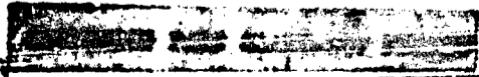
APPENDIX XI: Computer Program - The Stability of a Rotor with Timevarying Magnetic Forces

This appendix describes the computer program PN0351: "The Stability of a Rotor with Timevarying Magnetic Forces" and gives the detailed instructions for using the program. The program is based on the analysis contained in Appendix IX (and Appendix VIII) It calculates that value of the gradient of the timevarying magnetic force which is required to make the rotor unstable.

The rotor-bearing model is that of a general flexible rotor supported in a number of bearings (see Fig. 4). The dynamic bearing reactions are represented by 4 spring coefficients and 4 damping coefficients (see Volume 3). The rotor itself consists of a shaft whose diameter may vary in steps along the rotor and on which are fastened any number of masses (wheels, impellers, collars, etc.). At the centerplane of the alternator stator, timevarying magnetic forces act on the rotor. These forces may be both forces and moments and they are directly proportional to the rotor amplitudes and slopes. The forces depend strongly on the type of alternator and can be determined as discussed elsewhere in this report.

The stability computer program is not "automatic" in the sense that all that is required is to provide the numerical data describing the rotor and the magnetic forces, and then the program will calculate if the rotor is stable or not. It must definitely be emphasized that it requires judgement and several calculations to determine the rotor stability. A more detailed discussion of the problem is given in the text of the report and it is necessary to read that before attempting to use the program.

COMPUTER INPUT

An input data form is given in back of this appendix for quick reference when preparing the computer input. In the following the more detailed instructions are given.

Card 1 (72H) Any descriptive text may be given, identifying the calculation.

Card 2 (12I5) This is the "control card" whose values control the rest of the input. Thus, in order to understand some of the items it is necessary to refer to that particular part of the data to which the control number apply.

1.NS, specifies the number of rotor stations, see "Rotor Data" (NS ≤ 100).

2.NB, specifies the number of bearings (1 ≤ NB ≤ 10)

3.KA, The absolute value of KA, |KA|, specifies the number of that rotor station at which the magnetic forces are applied (i.e. at the centerplane of the alternator).

The program only provides for one such station.

When KA > 0 and KC ≥ 0 (see next item), there are only timevarying magnetic forces and no moments. When KA < 0 and KC ≥ 0, there are only timevarying magnetic moments and no forces. When KC ≤ -1, KA > 0 there are both timevarying magnetic forces and moments. For further details, see "Magnetic Force data."

4.KC KC is used to specify the form of the timevarying magnetic forces. When KC=0, the timevarying magnetic forces (KA > 0) or moments (KA < 0) depend only on the rotor amplitudes, not on the rotor slopes. When KC=1, the timevarying magnetic forces (KA > 0) or moments (KA < 0) depend only on the rotor slopes, not the rotor amplitudes. Finally, when KC=-1 there are both timevarying magnetic forces and moments, and they depend both on the rotor amplitudes and the rotor slopes. The reason for including this control parameter is to reduce the size of the stability determinant whenever possible. For further details, see "Magnetic Force Data."

5.NRP The program provides for including the effect of the bearing support pedestals whenever needed. If NRP=0, the bearing pedestals are rigid and no pedestal data is required in the input. Otherwise, set NRP=1 and specify the pedestal data.

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For further details, see "Pedestal Data."

6.NPD_ The bearings supporting the rotor may be either of the fixed geometry
type (plain cylindrical bearings, grooved sleeve bearings, ball bearings,
and so on) or they may be tilting pad bearings. If the bearings are of the
fixed geometry type, set NPD=0. If tilting pad bearings are employed, the
absolute value of NPD, $|NDP|$, specifies the number of pads in the bearing.
The bearing may be oriented such that the static bearing reaction goes between
the two bottom pads in which case NPD is positive and equal to the number of
pads. However, if the bearing is oriented such that the static bearing reaction
passes through the pivot of the bottom pad, NPD is negative and equal to minus
the number of bearing pads. The maximum value of $|NPD|$ is 8. For further
details, see "Bearing Data, Tilting Pad Bearing."

7.INC. The bearing lubricant may be incompressible (INC=0) as for oil bearings
or it may be compressible (INC=1) as for gas bearings. The difference in so
far as the program is concerned, is that the dynamic bearing coefficients with
a compressible lubricant are functions of the vibratory frequency which they
are not when the lubricant is incompressible. For further details, see "Bearing
Data."

8.NH For the stability calculation, the program evaluates the frequency response
of the rotor-bearing system. Theoretically, infinitely many frequencies are
required, but in practice only a limited number are necessary. These frequencies
are the half harmonics of the magnetic force frequency and NH specifies the
number of the highest half harmonic. NH must be equal to or greater than 2 but
cannot exceed 20. NH should not be made greater than necessary since the computer
time is almost proportional to the value of NH, and in many practical cases sufficient
accuracy is obtained by setting NH=2. A more detailed discussion is given in the
text. See also "Bearing Data."

9.NQ Each stability calculation is performed at a given rotor speed. With the speed
fixed the program varies the gradients of the timevarying magnetic forces over
a specified range to determine when instability is encountered. The variable

representing the magnetic force gradients is called Q_{ref}. If NQ=0, a range for Q_{ref} is employed, and it is necessary to specify the first and the last value of the range and also the increment by which the range should be covered. If NQ≥1, a list of NQ- values of Q_{ref} is given. For further details, see "Test Range of Magnetic Force Gradients."

10.NSP When it is desired to investigate several speed ranges for the same rotor and where either the bearing coefficients or the magnetic forces change from speed range to speed range, it is convenient not to have to repeat the "fixed" rotor data for each calculation. NSP gives the number of such speed ranges. The input data, starting from "Speed Data," must be repeated NSP times. NSP can be any positive value desired.

11. NDIA If NDIA=0, the program output will be limited to giving the value of the two stability determinants as a function of Q_{ref} for each rotor speed. In general, this should be adequate. However, in some cases it may be desired to study the behavior of the rotor in some detail. By setting NDIA=1, the computer output will also include the impedance matrices of the rotor at each of the specified frequencies and it can be investigated if any of the harmonics coincide with a resonance of the system. If NDIA=-1, the output will include not only the rotor impedance matrices but also the S_k-matrices employed in solving the determinants. (See Appendix IX).

12. INP If INP=0, the computer will expect to read in a completely new set of input data, starting from card 1, upon finishing the calculations for the present input data. Otherwise, set INP=1.

Card 3 (1P5E14.6)

1.YM YM gives Youngs modulus E for the shaft material in lbs/inch^2. If E actually changes along the rotor, it should be noted that the program only uses E in the produce EI where I is the cross-sectional moment of inertia of the shaft. Since $I = \frac{\pi}{64} (d_o^4 - d_i^4)$, where d_o is the outer shaft diameter and d_1 is the inner shaft diameter, any variation in E can be absorbed by changing d_o (see "Rotor Data") .

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2. DNST specifies the weight density of the shaft material, lbs/inch^3. The program converts it into the mass density ρ = DNST/386.069.

3. SHM gives the product αG where G is the shear modulus, lbs/inch^2, and α is the shape factor for shear (for circular cross-sections: $\alpha \cong 0.75$)

Rotor Data (8E9.2)

Referring to figure 4, the rotor is represented by a number of stations connected by shaft sections of uniform diameter. Thus, rotor stations are introduced wherever the shaft diameter changes (or changes significantly). Also, there must be a rotor station at each end of the rotor, at each bearing centerline and at the centerplane of the alternator where the magnetic forces are applied (Station KA).

Furthermore, a rotor station is introduced wherever the shaft has a concentrated mass which cannot readily be represented in terms of an inner and outer shaft diameter (impellers, turbine wheels, alternator poles, and so on). In this way the rotor is assigned a total of NS stations (card 2, item 1) which are numbered consecutively starting from one end of the rotor. There can be a maximum of 100 stations. Each station can be assigned a concentrated mass m with a polar mass moment of inertia I_p and a transverse mass moment of inertia I_T (any of these quantities may, of course, be zero). Also, each station can be assigned a shaft section with which it is connected to the following station. This shaft section has a length ℓ, an outer diameter $(d_o)_{stiff}$, an outer diameter $(d_o)_{mass}$ and an inner diameter d_i. The outer diameter $(d_o)_{stiff}$ is used to specify the stiffness of the shaft section such that the cross-sectional moment of inertia of the shaft is: $I = \frac{\pi}{64}\left[(d_o)_{stiff}^4 - d_i^4\right]$ and the shear area is: $\frac{\pi}{4}\left[(d_o)_{stiff}^2 - d_i^2\right]$. The outer diameter $(d_o)_{mass}$ is used in calculating the mass of the shaft such that the mass per unit length is: $\rho \cdot \frac{\pi}{4}\left[(d_o)_{mass}^2 - d_i^2\right]$ where ρ is the mass density (see card 3, item 2).

In the computer input there must be a card for each rotor station (NS cards). Each card specifies the 7 values for the station:

1. The concentrated mass: m, lbs. (may be zero)

2. The polar mass moment of inertia of the station mass; I_p lbs-inch^2 (may be zero).

3. The transverse mass moment of inertia of the station mass; I_T lbs-inch^2 (may be zero).

4. The length of the shaft section to the next station: ℓ , inch. (may be zero). For the last station, set $\ell = 0$.

5. The outer diameter, $(d_o)_{stiff}$ of the shaft section, inch. $(d_o)_{stiff}$ is used in calculating the stiffness of the shaft section; $(d_o)_{stiff} \neq 0$. For the last station, set $(d_o)_{stiff} = 1.0$.

6. The outer diameter, $(d_o)_{mass}$ of the shaft section, inch. (may be zero). $(d_o)_{mass}$ is used in calculating the mass of the shaft section. For the last station, set $(d_o)_{mass} = 0$.

7. The inner diamter, d_i of the shaft section, inch (may be zero). d_i is used both in calculating the stiffness and the mass of the shaft section. For the last station, set $d_i = 0$.

Bearing Stations (12I5)

The rotor bearing station numbers at which there are bearings, are listed in sequence. There can be up to 10 bearings.

Pedestal Data (8E9.2)

The program provides for the option that the pedestals supporting the bearings may be flexible. In that case, data for the pedestals must be given and NRP must be set equal to 1 (card 2, item 5). If the pedestals are rigid, set NRP=0 and omit giving any data for the pedestals.

When NRP=1, each bearing is supported in a "two-dimensional" pedestal. The pedestal is represented as two separate masses, each mass on its own spring and dashpot. The one mass-spring-dashpot system represents the pedestal characteristics in the x-direction,(the vertical direction) and the other system represents the y-direction (the horizontal direction). There is no coupling between the two systems. In the computer input there must be one card for each rotor bearing which gives the 6 items necessary to specify the pedestal characteristics:

1. The pedestal mass for the x-direction, lbs.

2. The pedestal stiffness for the x-direction, lbs/inch

3. The pedestal damping coefficient for the x-direction, lbs-sec/inch

(note: for the bearing films the damping is given in lbs/inch, whereas the damping coefficient in lbs-sec/inch is used for the pedestals)

4. The pedestal mass for the y-direction, lbs.

5. The pedestal stiffness for the y-direction, lbs/inch

6. The pedestal damping coefficient for the y-direction, lbs-sec/inch.

Speed Data (1P5E14.6)

Usually it is desired to make calculations not just for a single rotor speed but for a range of speeds. Even though the bearing coefficients and also the magnetic forces are somewhat dependent on speed, it is convenient to be able to perform calculations for several speeds without having to change the bearing coefficient data and the magnetic force data. The present input card allows specifying such a speed range by giving the first speed and the last speed of the range and the increment by which the range should be covered. If it is desired to run only one speed, let the initial speed be equal to the desired speed and let the final speed be less than this value whereas the speed increment is set equal to zero. The speed data card also specifies the ratio between the frequency of the timevarying magnetic forces and the speed of the rotor. For the 4 pole homopolar generator, this ratio is equal to 2, and for the heteropolar generator under load, the ratio is equal

to the number of rotor teeth. Finally, the speed data card also specifies the scalefactor for the stability determinants. Hence, in total the speed data card has five values:

1. Initial speed of speed range, rpm

2. Final speed of speed range, rpm

3. Increment by which the speed range is covered, rpm

4. Ratio between magnetic force frequency and rotor speed (=2 for the 4 pole homopolar generator, and equal to the number of rotor teeth for the heteropolar generator under load)

5. Scalefactor for the stability determinants. In order to control computer overflow, each element of the two stability determinants (i.e. for even and odd indicies) is divided by the product of the specified scalefactor and the square of the angular rotor speed. It is recommended to set the scalefactor equal to the rotor mass (in lbs-sec^2/in) times approximately 1/8 the square of the product of the highest harmonic and the magnetic force frequency ratio. However, the choice of scalefactor in no way influences the accuracy of the calculations and if in doubt, set the scalefactor equal to 1. If overflow is encountered, increase the scalefactor.

Magnetic Force Data

The generator magnetic forces are made up of two parts: a static component and a timevarying component. Let the rotor amplitudes at the centerplane of the alternator be x and y, and let the corresponding slopes of the rotor be Θ and φ (i.e. $\Theta = \frac{dx}{dz}$, $\varphi = \frac{dy}{dz}$ where z is the coordinate along the rotor). Then the magnetic forces and moments acting on the rotor are proportional to x, y, Θ and φ . The static components can be written:

static magnetic forces
$$\begin{cases} F_x = Q_0 x \\ F_y = Q_0 y \end{cases}$$

static magnetic moments
$$\begin{cases} T_x = Q_0' \Theta \\ T_y = Q_0' \varphi \end{cases}$$

where Q_0 is the negative radial stiffness in lbs/inch and Q_0' is the negative angular stiffness in lbs-inch/radian.

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The first card of the magnetic force data specify the two negative static stiffnesses.

1. The negative of the radial static stiffness Q_0 , lbs/inch

2. The negative of the angular static stiffness Q_0' , lbs-inch/radian

Q_0 and Q_0' are positive values, and the program assumes them to act as negative springs. For the heteropolar generator, $Q_0' = 0$.

This card is followed by several cards speficying the gradients of the time-varying magnetic forces and moments. These forces and moments are written:

$$
\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = -Q_{ref} \left[\begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{x\theta} & Q_{x\varphi} \\ Q_{yx} & Q_{yy} & Q_{y\theta} & Q_{y\varphi} \\ Q_{\theta x} & Q_{\theta y} & Q_{\theta\theta} & Q_{\theta\varphi} \\ Q_{\varphi x} & Q_{\varphi y} & Q_{\varphi\theta} & Q_{\varphi\varphi} \end{Bmatrix} \cos(\Omega t) - \begin{Bmatrix} q_{xx} & q_{xy} & q_{x\theta} & q_{x\varphi} \\ q_{yx} & q_{yy} & q_{y\theta} & q_{y\varphi} \\ q_{\theta x} & q_{\theta y} & q_{\theta\theta} & q_{\theta\varphi} \\ q_{\varphi x} & q_{\varphi y} & q_{\varphi\theta} & q_{\varphi\varphi} \end{Bmatrix} \sin(\Omega t) \right] \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix} \quad (L.1)
$$

The units of the gradients are:

$\quad Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy} ; q_{xx}, q_{xy}, q_{yx}, q_{yy} \quad$ in $\quad$ lbs/inch

$\quad Q_{x\theta}, Q_{x\varphi}, Q_{y\theta}, Q_{y\varphi}, q_{x\theta}, q_{x\varphi}, q_{y\theta}, q_{y\varphi} \quad$ in $\quad$ lbs/radian

$\quad Q_{\theta x}, Q_{\theta y}, Q_{\varphi x}, Q_{\varphi y}, q_{\theta x}, q_{\theta y}, q_{\varphi x}, q_{\varphi y} \quad$ in $\quad$ lbs·inch/inch

$\quad Q_{\theta\theta}, Q_{\theta\varphi}, Q_{\varphi\theta}, Q_{\varphi\varphi}, q_{\theta\theta}, q_{\theta\varphi}, q_{\varphi\theta}, q_{\varphi\varphi} \quad$ in $\quad$ lbs·inch/radian

The program searches for the threshold of instability by varying a reference value Q_{ref}, representing the gradients of the timevarying magnetic forces (see the above equation), but the matrices themselves remain unchanged in the search operation. Thus, when $Q_{ref}=1$, the actual operating conditions of the alternator is encountered (or, in general, when $Q_{ref} \cdot (Q_{xx})_{input}$, etc. is

equal to the actual value.)

The program provides for five possibilities: either give the full mat__ __es as
shown, or any of the four submatrices. Which option to use depends on _he generato_
type, and it is specified by means of the two control numbers on card __ :
KC (item 4) and the sign of KA (item 3).

For the 4 pole homopolar generator or the two-coil Lundell generator w__ re the
north poles and the southpoles are in separate planes, the complete fo_ _y four
matrices are used (see Appendix I). In that case, set KC=-1 and let K_ _e positive
The input _onsists of 8 cards with 4 values per card:

(1P4E14.6)

$$KC=-1, \quad KA > 0$$

$$
\begin{array}{cccc}
Q_{xx} & Q_{xy} & Q_{x\theta} & Q_{x\varphi} \\
Q_{yx} & Q_{yy} & Q_{y\theta} & Q_{y\varphi} \\
Q_{\theta x} & Q_{\theta y} & Q_{\theta\theta} & Q_{\theta\varphi} \\
Q_{\varphi x} & Q_{\varphi y} & Q_{\varphi\theta} & Q_{\varphi\varphi} \\
q_{xx} & q_{xy} & q_{x\theta} & q_{x\varphi} \\
q_{yx} & q_{yy} & q_{y\theta} & q_{y\varphi} \\
q_{\theta x} & q_{\theta y} & q_{\theta\theta} & q_{\theta\varphi} \\
q_{\varphi x} & q_{\varphi y} & q_{\varphi\theta} & q_{\varphi\varphi}
\end{array}
$$

For the 4 pole homopolar generator it is found by the present analysis _ _t
$Q_{x\theta} = -Q_{y\varphi} = Q_{\theta x} = -Q_{\varphi y} = -q_{x\varphi} = -q_{y\theta} = -q_{\theta y} = -q_{\varphi x}$ whereas the remaining gradie _ _ are zero.

For the heteropolar generator und_ _r load there are only timevarying for _ and
no moments, i.e. only Q_{xx}, Q_{yy}, q_{xx} and q_{yy} are different from zero. Fur _rmore,
$Q_{xx} = Q_{yy}$ and $q_{xx} = q_{yy}$. In that case, set KC=0 and let KA be positive, _ _ give
4 cards with two values per card:

(1P4E14.6)   KC=0, KA > 0

$$Q_{xx} \quad Q_{xy}$$
$$Q_{yx} \quad Q_{yy}$$
$$q_{xx} \quad q_{xy}$$
$$q_{yx} \quad q_{yy}$$

To increase the versatility of the program, there are three additional possi-
bilities. If the timevarying magnetic forces are proportional to the slopes
of the rotor and there are no moments, set KC=1 and let KA be positive,
and give 4 cards with 2 values per card:

(1P4E14.6)

         KC=1, KA > 0

$$Q_{x\theta} \quad Q_{x\varphi}$$
$$Q_{y\theta} \quad Q_{y\varphi}$$
$$q_{x\theta} \quad q_{x\varphi}$$
$$q_{y\theta} \quad q_{y\varphi}$$

If the timevarying magnetic moments are proportional to the rotor amplitudes
and there are no forces, set KC=0 and let KA be negative, and give 4 cards with
2 values per card:

(1P4E14.6)

         KC=0, KA < 0

$$Q_{\theta x} \quad Q_{\theta y}$$
$$Q_{\varphi x} \quad Q_{\varphi y}$$
$$q_{\theta x} \quad q_{\theta y}$$
$$q_{\varphi x} \quad q_{\varphi y}$$

Finally, if the timevarying magnetic moments are proportional to the rotor
slopes and there are no forces, set KC=1 and let KA be negative, and give 4
cards with 2 values per card:

(1P4E14.6)

         KC=1, KA < 0

$$Q_{\theta\theta} \quad Q_{\theta\varphi}$$
$$Q_{\varphi\theta} \quad Q_{\varphi\varphi}$$
$$q_{\theta\theta} \quad q_{\theta\varphi}$$
$$q_{\varphi\theta} \quad q_{\varphi\varphi}$$

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Test Range of Magnetic Force Gradients

As explained under "Magnetic Force Data" in connection with eq. (L.1), the program searches for the threshold of instability as the zero points of two determinants by varying a parameter, Q_{ref}, which represents the gradients of the timevarying magnetic forces and moments. The actual gradients are equal to their respective input values times Q_{ref}. Thus, the input values for the gradients can be set equal to any value proportional to the actual gradients as long as the proportionality factor is the same for all the gradients, and when Q_{ref} equals this proportionality factor, the actual operating condition of the generator is encountered. In performing the stability calculation, Q_{ref} should in general be allowed a much wider range than the one corresponding to the actual generator operating condition. The range of Q_{ref} can be specified in two ways in the input. If NQ=0 (card 2, item 9) an initial value and a final value of Q_{ref} is specified together with an increment by which the program covers the specified range. Give one card with 3 values:

(1P5E14.6)

1. Initial value of Q_{ref}

2. Final value of Q_{ref}

3. Increment of Q_{ref}

If NQ $\geq$ 1 (card 2, item 9), the stability determinants are evaluated at specified values of Q_{ref}. Give a total of NQ-values or Q_{ref} with 5 values per card according to the format (1P5E14.6).

Bearing Data, Fixed Geometry

When the bearings supporting the rotor are not of the tilting pad type, set NPD=0 (card 2, item 6). Then the dynamic reaction for each bearing is represented in terms of 8 coefficients. In other words, introduce a fixed x-y-coordinate system with origin in the steady-state position of the journal center, and let the corresponding journal amplitudes be x and y. Then the

bearing reactions F_x and F_y are:

$$F_x = -K_{xx}\, x - B_{xx}\frac{dx}{dt} - K_{xy}\, y - B_{xy}\frac{dy}{dt}$$

$$F_y = -K_{yx}\, x - B_{yx}\frac{dx}{dt} - K_{yy}\, y - B_{yy}\frac{dy}{dt} \qquad (L.2)$$

where the K's represent the bearing film stiffness and the B's represent the damping. Values for typical bearings are given in Volume 3 and Volume 4.

If the bearing lubricant is incompressible (INC=0, card 2, item 7), there must be one card for each bearing (i.e. a total of NB cards, see card 2, Item 2). On each card are 8 values:

(8E9.2)

1. K_{xx} = spring coefficient, lbs/inch

2. ωB_{xx} = damping, lbs/inch

3. K_{xy} = Spring coefficient, lbs/inch

4. ωB_{xy} = Damping, lbs/inch

5. K_{yx} = spring coefficient, lbs/inch

6. ωB_{yx} = damping, lbs/inch

7. K_{yy} = spring coefficient, lbs/inch

8. ωB_{yy} = damping, lbs/inch.

Here, ω is the angular speed of the rotor in radians/sec., or, in other words, the four input values for damping gives the total damping at one per revolution, not the damping coefficients. This is in accordance with the way these coefficients are calculated from lubrication theory (see Volume 3).

If the bearing lubricant is a gas and, therefore, compressible, set INC=1 (card 2, item 7). Then the 8 bearing coefficients become functions of that frequency, ν , with which the rotor vibrates (the effect of squeeze number.) In calculating the stability determinants, the program calculates the rotors frequency response for as many frequencies as specified by the number of harmonics and, thus, it is necessary in the input to give the 8 bearing coefficients at

these frequencies. Let the timevarying magnetic forces have the frequency Ω radians/sec (the ratio $\frac{\Omega}{\omega}$ is given on the "Speed Data" card, item 4). Then it is necessary to evaluate the 8 bearing coefficients for (NH+1)-frequencies (NH is given on card 2, item 8). These frequencies, ν, are:

$$\frac{\nu}{\omega} = 0 \,,\, \tfrac{1}{2}\left(\tfrac{\Omega}{\omega}\right),\, \left(\tfrac{\Omega}{\omega}\right),\, \tfrac{3}{2}\left(\tfrac{\Omega}{\omega}\right),\, 2\left(\tfrac{\Omega}{\omega}\right),\, - \,-\,-\,,\, \frac{NH}{2}\left(\tfrac{\Omega}{\omega}\right) \qquad (L.3)$$

where $\frac{\Omega}{\omega}$ is given by item 4 on the "Speed Data" card. It should be noted, that when ν is different from ω, the "ω" in the four damping values, $\omega B_{xx}\,,\, \omega B_{xy}\,,\, \omega B_{yx}$ and ωB_{yy}, is still the angular speed of the rotor.

Hence, for a compressible lubricant give (NH+1)-cards per bearing where each card contains the values of the 8 bearing coefficients according to the same format as given above for an incompressible lubricant. The first card is for $\frac{\nu}{\omega} = 0$ and the last card for $\frac{\nu}{\omega} = \frac{NH}{2}\left(\tfrac{\Omega}{\omega}\right)$.

In this way, there will be a total of NB · (NH+1) cards with data for the bearing coefficients.

Bearing Data, Tilting Pad Bearing

When the bearings supporting the rotor are tilting pad bearings, set NPD equal to plus or minus the number of pads (card 2, item 6). The program assumes that the pads are arranged symmetrical with respect to a vertical axis so that the bearing operates with zero attitude angle. Hence, pads opposite each other operate under the same conditions and have the same dynamic coefficients, and it would be superflous to repeat the same input for two pads. The program is set up to avoid such repetition of input. Furthermore, the tilting pad bearing may be oriented in two ways. Either the static bearing load passes between the two bottom pads or the load passes through the pivot of the bottom pad. In the first case, set NPD equal to the total number of pads. In the second case, set NPD equal to minus the total number of pads (i.e. the number of pads is equal to |NPD|). For each bearing the program requires input for NPD1 number of pads:

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If NPD is positive (load between pads): $\begin{cases} \text{NPD even, NPD1=1/2 NPD} \\ \text{NPD odd, NPD1=1/2·(NPD+1)} \end{cases}$

If NPD is negative (load on pad): $\begin{cases} \text{NPD even, NPD1=1/2·|NPD|+1} \\ \text{NPD odd, NPD1=1/2·(|NPD|+1)} \end{cases}$

Thus, for a four shoe bearing where the pivots are 45 degrees from the vertical load line, NPD=4 and NPD1=2. For a three shoe bearing where the vertical load line passes through the pivot of the bottom pad, NPD=-3 and NPD1=2.

Each pad film is represented by 8 dynamic coefficients as defined by eq. (L.2). However, here the x-axis passes through the pivot and the y-axis is perpendicular to the x-axis. The origin of the x-y-system changes from pad to pad.

For each bearing there must be data for NPD1 pads. The first card for a pad specifies the mass moment of inertia of the pad, the mass of the pad, the radial stiffness of the pivot support and the angle from the vertical load line to the pivot point:

(1P5E14.6)

1. The mass moment inertia of the pad with respect to the pitch axis divided by the square of the journal radius, lbs. The pitch axis is the axis parallel to the rotor axis through the pivot point.

2. The mass of the pad, lbs.

3. The radial stiffness of the pivot and its support, lbs/inch.

4. The angle from the static bearing load line to the pivot of the pad, degrees.

Then follows a card with the 8 pad film coefficients:

(8E9.2)

1. K_{xx} lbs/inch

2. ωB_{xx} lbs/inch

3. K_{xy} lbs/inch

4. ωB_{xy} lbs/inch

5. K_{yx} lbs/inch

6. ωB_{yx} lbs/inch

7. K_{yy} lbs/inch

8. ωB_{yy} lbs/inch

If the lubricant is incompressible (i.e. INC=0, card 2, item 7), there is only
one card per pad with · bearing coefficients. However, if the lubricant is
compressible (INC=1), there must be (NH+1)-cards per pad with coefficients
(for explanation, see "Bearing Data, Fixed Geometry"). Thus, for each pad there
are either 2 input cards or (NH+2) input cards. Since the program requires data
for NPD1 pads, there are either 2·NPD1 or (NH+2)·NPD1 cards per bearing. With
NB bearings, the total bearing data input requires 2·NPD1·NB cards if INC=0,
or (NH+2)·NPD1·NB cards if INC=1.

COMPUTER OUTPUT

Referring to the later given calculation where the output from the computer is
shown, it is seen that the program output denotes the first couple of pages to
a listing of the input values. Thereby any errors in the input are readily
spotted. The input values are listed in the same sequence as the one in which
they are given to the program. The only input data which are not repeated, is
the card specifying the speed data and the card (or cards) specifying the test
range of the magnetic force gradients.

After the listing of the input data follow the results of the calculations.
For each rotor speed there will be a 3-column list. The first column lists the
reference values of the magnetic force gradient (i.e. Q_{ref}). It is labeled
"QXX" in the output. For each value of Q_{ref}, the values of the two stability
determinants are given. The first determinant, labeled "EVEN DETERM.", is the
stability determinant for even indices (see eq. (J.25), Appendix IX), and the
second determinant, labeled "ODD DETERM." is the determinant for odd indices (see
eq. (J.28), Appendix IX). Usually, the odd determinant is the one of greatest
interest. It is the one that defines the instability zones centered at

$$\frac{\Omega}{\omega_{crit}} = 2, \frac{3}{3}, \frac{2}{5}, \frac{2}{7}, - - - \quad (\Omega \text{ is the frequency of the magnetic forces and}$$

$\omega_{critical}$ represents the critical speeds of the rotor-bearing system). If the speed of the rotor is ω, these instability zones are centered at:

$$\omega = \frac{\omega_{critical}}{\left(\frac{\Omega}{\omega}\right)} \left\{ \begin{array}{c} 2 \\ \frac{3}{3} \\ \frac{5}{2} \\ \frac{7}{2} \\ | \end{array} \right\}$$

where $\left(\frac{\Omega}{\omega}\right)$ is the fixed ratio between the magnetic force frequency and the rotor speed, specified in the input (see Item 4, "Speed Data"). Of these instability zones, the first one is by far the most important except in very unusual circumstances.

The even determinant defines the instability zones centered at:

$$\omega = \frac{\omega_{critical}}{\left(\frac{\Omega}{\omega}\right)} \left\{ \begin{array}{c} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ | \end{array} \right\}$$

Whenever one of the two determinants is zero, the corresponding value of Q_{ref} defines a point on a boundary between a stable zone of operation and an unstable zone. The results do not indicate on which side of this boundary the system is stable or unstable. Hence it is necessary to perform calculations at sufficiently many rotor speeds to make it possible to draw up a stability map in the neighborhood of the operating speed.

SAMPLE CALCULATION

To illustrate the use of the stability computer program, a four pole homopolar generator with a turbine drive has been examined for stability. The rotor bearings are gas lubricated, and with a bearing stiffness of approximately 200,000 lbs/inch. The first three critical speeds are at 14,700 rpm, 16,000 rpm and 34,630 rpm. Because of the change in stiffness with frequency, these critical speeds are not the same for all harmonics. Calculations are performed over a speed range of

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10,300 rpm to 20,300 rpm in increments of 1,000 rpm and the magnetic force gradient ranges from 0 to 1,000,000 in increments of 10,000. The resulting stability map is shown in Figure 20 . Three stability boundaries are well defined, labeled 1, 2 and 3, respectively. Boundaries 1 and 2 derive from zero-points of the odd determinant and are centered around the first and the second critical speed such that the rotor is unstable for operation between 15,200 rpm and 18,000 rpm. At the boundaries, the determinant actually crosses zero and the program then automatically interpolates to find the accurate value of Q_{ref} at which the determinant becomes zero. Boundary No. 3 derives from zero points of the even determinant. In the output this determinant is never exactly zero but it is readily seen that the determinant has a minimum point whose value is equal to zero considering the numerical accuracy of the computation. In addition, discrete points of another boundary, labeled 4 in the map, have been obtained at 10,300 rpm, 11,300 rpm and 12,300 rpm. They derive from the zero-points of the odd determinant (the determinant has a minimum at these points). They are probably induced by excitation of 1/3 of the third critical speed in which case they would define an instability zone centered at 11,540 rpm, but more detailed calculations are needed to obtain a closer definition of this zone.

When the bearings are assigned their proper damping values it will be found that all the stability boundaries move upwards in the stability map such that, as an example, the two branches of boundary 1 meet and no longer reach the abscissa axis. However, if the rotors operating speed is within any of the indicated instability zones, although the bearing damping may stabilize the rotor, the stability margin must be considered small, and even if the rotor is not exactly unstable, the system is "weak" in the same sense as a system operating at its natural frequency whose amplitude is controlled solely by the damping available in the system. Therefore, operation within the instability zones indicated in Fig. 20   should be avoided.

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INPUT FORM FOR COMPUTER PROGRAM
PN0351: THE STABILITY OF A ROTOR WITH TIMEVARYING MAGNETIC FORCES

Card 1 (72H)   Text

Card 2 (1215)

1. NS = Number of rotor stations (NS ≤ 10)
2. NB = Number of bearings (NB ≤ 10)
3. KA |KA|=Rotor station number at which magnetic forces act

   KA > 0: forces only, no moments ⎫

   KA < 0: moments only, no forces ⎬ KC ≥ 0

   KA > 0, KC=-1: both forces and moments
4. KC   KC=0: the magnetic forces or moments are proportional to amplitudes

   KC=1: the magnetic forces or moments are proportional to slope

   KC=-1: there are both magnetic forces and moments
5. NRP NRP=0: bearing pedestals are rigid, no pedestal input data

   NRP=1: flexible bearing pedestals, pedestal input data required
6. NPD NPD=0: fixed geometry bearings

   NPD ≥ 1: number of pads in tilting pad bearing, load between pads

   NPD ≤ -1: |NPD|=number of pads in tilting pad bearing, load on pad
7. INC INC=0: bearing lubricant is incompressible

   INC=1: bearing lubricant is compressible
8. NH = Number of frequency harmonics in stability calculation (2 ≤ NH ≤ 20)
9. NQ   NQ=0: give range of Q_{ref}, program increments

   NQ ≥ 1: give NQ-values of Q_{ref}
10. NSP = Number of speed ranges with accompanying data (NSP ≥ 1)
11. NDIA NDIA=0: rotor impedance matrices not included in output

    NDIA=1: rotor impedance matrices included in output

    NDIA=-1: diagnostic
12. INP INP=0: more input follows, starting from card 1

    INP=1: last set of input data

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Card 3 (1P5E14.6)

1. YM = Youngs modulus for shaft material, lbs/in^2
2. DNST = Weight density of shaft material, lbs/in^3
3. SHM = α G, where G is shear modulus, lbs/in^2, and α is shape factor for shear.

Rotor Data (8E9.2)
Give NS cards with 7 numbers on each card:

1. Mass at rotor station, lbs.
2. Polar mass moment of inertia at rotor station, lbs-in^2
3. Transverse mass moment of inertia at rotor station, lbs-in^2
4. Length of shaft section to next station, inch
5. Outer shaft diameter for cross-sectional moment of inertia, inch
6. Outer shaft diameter for shaft mass, inch
7. Inner shaft diameter, inch.

Bearing Stations (12I5)

List the rotor stations at which there are bearings, in total NB stations

Pedestal Data (8E9.2)

This data only applies when NRP=1 (card 2, item 5). Give a total of NB cards with 6 values per card:

1. Pedestal mass, x-direction, lbs.
2. Pedestal stiffness, x-direction, lbs/inch
3. Pedestal damping, x-direction, lbs-sec/inch
4. Pedestal mass, y-direction, lbs.
5. Pedestal stiffness, y-direction, lbs/inch
6. Pedestal damping, y-direction, lbs-sec/inch

Note: The following data must be repeated NSP-times (Card 2, Item 10)

<u>Speed Data (1P5E14.6)</u>

Give one card with 5 values:

1. Initial speed, rpm
2. Final speed, rpm
3. Speed increment, rpm
4. Ratio of magnetic force frequency to rotor speed
5. Scale factor for determinant (set equal to mass of rotor)

<u>Magnetic Force Data</u>

<u>Card (1P5E14.6)</u>

1. Static gradient of magnetic force, Q_0 , lbs/in
2. Static gradient of magnetic moment, Q_0' , lbs-inch/radian

<u>Cards (1P4E14.6)</u>

a. If KC=-1 (card 2, item 4), give 8 cards with 4 values per card:

$$
\begin{array}{cccc}
Q_{xx} & Q_{xy} & Q_{x\theta} & Q_{x\varphi} \\
Q_{yx} & Q_{yy} & Q_{y\theta} & Q_{y\varphi} \\
Q_{\theta x} & Q_{\theta y} & Q_{\theta\theta} & Q_{\theta\varphi} \\
Q_{\varphi x} & Q_{\varphi y} & Q_{\varphi\theta} & Q_{\varphi\varphi} \\
q_{xx} & q_{xy} & q_{x\theta} & q_{x\varphi} \\
q_{yx} & q_{yy} & q_{y\theta} & q_{y\varphi} \\
q_{\theta x} & q_{\theta y} & q_{\theta\theta} & q_{\theta\varphi} \\
q_{\varphi x} & q_{\varphi y} & q_{\varphi\theta} & q_{\varphi\varphi}
\end{array}
$$

These are the gradients of the timevarying magnetic forces and moments:

$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}, q_{xx}, q_{xy}, q_{yx}, q_{yy}$ in lbs/inch

$Q_{x\theta}, Q_{x\varphi}, Q_{y\theta}, Q_{y\varphi}, q_{x\theta}, q_{x\varphi}, q_{y\theta}, q_{y\varphi}$ in lbs/radian

$Q_{\theta x}, Q_{\theta y}, Q_{\varphi x}, Q_{\varphi y}, q_{\theta x}, q_{\theta y}, q_{\varphi x}, q_{\varphi y}$ in lbs-inch/inch

$Q_{\theta\theta}, Q_{\theta\varphi}, Q_{\varphi\theta}, Q_{\varphi\varphi}, q_{\theta\theta}, q_{\theta\varphi}, q_{\varphi\theta}, q_{\varphi\varphi}$ in lbs-inch/radian

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b. If KC=0, give 4 cards with 2 values per card

<table>
<tr><td colspan="2">$\underline{KA > 0}$</td><td colspan="2">$\underline{KA < 0}$</td></tr>
<tr><td>Q_{xx}</td><td>Q_{xy}</td><td>$Q_{\theta x}$</td><td>$Q_{\theta y}$</td></tr>
<tr><td>Q_{yx}</td><td>Q_{yy}</td><td>$Q_{\varphi x}$</td><td>$Q_{\varphi y}$</td></tr>
<tr><td>q_{xx}</td><td>q_{xy}</td><td>$q_{\theta x}$</td><td>$q_{\theta y}$</td></tr>
<tr><td>q_{yx}</td><td>q_{yy}</td><td>$q_{\varphi x}$</td><td>$q_{\varphi y}$</td></tr>
</table>

c. If KC=1, give 4 cards with 2 values per card

<table>
<tr><td colspan="2">$\underline{KA > 0}$</td><td colspan="2">$\underline{KA < 0}$</td></tr>
<tr><td>$Q_{x\theta}$</td><td>$Q_{x\varphi}$</td><td>$Q_{\theta\theta}$</td><td>$Q_{\theta\varphi}$</td></tr>
<tr><td>$Q_{y\theta}$</td><td>$Q_{y\varphi}$</td><td>$Q_{\varphi\theta}$</td><td>$Q_{\varphi\varphi}$</td></tr>
<tr><td>$q_{x\theta}$</td><td>$q_{x\varphi}$</td><td>$q_{\theta\theta}$</td><td>$q_{\theta\varphi}$</td></tr>
<tr><td>$q_{y\theta}$</td><td>$q_{y\varphi}$</td><td>$q_{\varphi\theta}$</td><td>$q_{\varphi\varphi}$</td></tr>
</table>

Test Range of Magnetic Force Gradients (1P5E14.6)

a. If NQ=0 (card 2, item 9): Give 1 card with 3 values:

   1. Initial value of Q_{ref}

   2. Final value of Q_{ref}

   3. Increment of Q_{ref}

b. If NQ $\geq$ 1: Give cards with 5 values of Q_{ref} per card, total NQ-values

Bearing Data, Fixed Geometry (8E9.2)

Applies when NPD=0 (card 2, item 6). If the lubricant is incompressible (INC=0; card 2, item 7), give one card per bearing. If the lubricant is compressible (INC=1), give (NH+1)-cards per bearing (NH is item 8, card2). Each card gives a set of 8 bearing coefficients:

   1. Spring coefficient K_{xx}, lbs/inch

   2. Damping ωB_{xx} , lbs/inch

   3. Spring coefficient K_{xy}, lbs/inch

   4. Damping ωB_{xy} , lbs/inch

5. Spring coefficient K_{yx}, lbs/inch
6. Damping ωB_{yx}, lbs/inch
7. Spring coefficient K_{yy}, lbs/inch
8. Damping ωB_{yy}, lbs/inch

Bearing Data, Tilting Pad Bearing

Applies when NPD $\neq$ 0 (card 2, item 6). Define the number NPD1 by:

if NPD $\geq$ 1 (load between pads): $\begin{cases} \text{NPD even, then:} & \text{NPD1=1/2·NPD} \\ \text{NPD odd, then:} & \text{NPD1=1/2·(NPD+1)} \end{cases}$

if NPD $\leq$ -1 (load on pad): $\begin{cases} |\text{NPD}| \text{ even, then:} & \text{NPD1=1/2·}|\text{NPD}|\text{+1} \\ |\text{NPD}| \text{ odd, then:} & \text{NPD1=1/2·}(|\text{NPD}|\text{+1}) \end{cases}$

NPD1 is the number of pads for which input is required per bearing. If the lubricant is incompressible (INC=0; card 2, item 7), give two cards per pad. If the lubricant is compressible (INC=1), give (NH+2)-cards per pad. In either case the first card is:

(1P5E14.6)

1. Pitch mass moment of inertia divided by the square of the journal radius, lbs
2. Pad mass, lbs
3. Radial stiffness of pivot support, lbs/inch
4. Angle from bearing load line to pivot point, degrees.

Then follow 1 card if INC=0, or (NH+1)-cards if INC=1, with the 8 dynamic coefficients for the pad:

Cards (8E9.2)

1. Spring coefficient K_{xx}, lbs/inch
2. Damping ωB_{xx}, lbs/inch
3. Spring coefficient K_{xy}, lbs/inch
4. Damping ωB_{xy}, lbs/inch

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5. Spring coefficient K_{yx}, lbs/inch
6. Damping ωB_{yx}, lbs/inch
7. Spring coefficient K_{yy}, lbs/inch
8. Damping ωB_{yy}, lbs/inch

These (NH+2)-cards must be repeated NPD1 times per bearing, and there must be one complete set for each bearing (there are NB bearings).

```
C     3-14-67    J.LUND    MECHANICAL TECHNOLOGY INC.
C     PN351=ROTOR STABILITY WITH MAGNECTIC FORCES
      DIMENSION  RM(10C),RIP(100),RL(1CO),RS(100),RW(100),RD(100),
     1DVXA(100),DVXB(100),DVXC(100),DVXD(100),DVYA(100),DVYB(100),
     2DVYC(100),DVYC(1CO),DVUX(100),DVUY(100),DMUX(100),DMUY(100),
     3B1(100),B2(100),B3(100),B4(100),B5(100),B6(100),B7(100),B8(100),
     4B9(100),B10(100),PMX(10),PKX(1C),PDX(10),PMY(10),PKY(10),PDY(10),
     5SXX(10),DXX(10),SXY(10),DXY(10),SYX(10),DYX(10),SYY(10),DYY(10),
     6QLST(200),LB(10),CMXA(100),RIT(100)
      DIMENSION  BKXX(10,21),BCXX(10,21),BKXY(10,21),BCXY(10,21),
     1BKYX(10,21),BCYX(10,21),BKYY(10,21),BCYY(10,21),PMIN(10,5),
     2PADM(10,5),PACK(10,5),PANG(10,5),DEVN(8,8),DEDD(8,8),CMR(4,8),
     3CME(4,8),AMR(8,8),AME(8,8),WR(8,8),WE(8,8),WA(8,8),WB(4,4),
     4WCC(8,1),UR(8,8),UE(8,8),EMR(4,4),EME(4,4),WC(4,4),WSQ(4,4)
      DIMENSION  PKXX(10,5,21),PCXX(10,5,21),PKXY(10,5,21),
     1PCXY(10,5,21),PKYX(10,5,21),PCYX(10,5,21),PKYY(10,5,21),
     2PCYY(10,5,21),GR(4,4,21),GE(4,4,21)
      WRITE (6,99)                                                      1
  190 READ (5,100)                                                      2
      READ (5,101) NS,.NB, KA, KC, NRP, NPD, INC, NH, NQ, NSP, NDIA, INP  3
      READ (5,102) YM, CNST, SHM                                        16
      WRITE(6,100)                                                      17
      WRITE(6,103)                                                      18
      WRITE(6,104)NS,NB,KA,KC,NRP,NPD,INC,NH,NQ,NSP,NDIA,INP            19
      WRITE(6,105)                                                      20
      WRITE(6,102)YM,DNST,SHM                                           21
      DNST=CNST/386.069
      NS1=NS-1
      NH1=NH+1
      IF(KC) 196,195,195
  195 KQ1=4
      KQ2=2
      KQ3=6
      GO TO 197
  196 KQ1=8
      KQ2=4
      KQ3=8
  197 IF(KA) 198,199,199
  198 KB=-KA
      GO TO 200
  199 KB=KA
  200 WRITE(6,110)                                                      37
      WRITE(6,108)                                                      38
      DO 203 J=1,NS
      READ (5,106) RM(J), RIP(J), RIT(J), RL(J), RS(J), RW(J), RD(J)    41
      WRITE(6,107)J,RM(J),RIP(J),RIT(J),RL(J),RS(J),RW(J),RD(J)         49
      RM(J)=RM(J)/386.069
      RIP(J)=RIP(J)/386.069
      RIT(J)=RIT(J)/386.069
      C1=0.049087385*YM*(RS(J)**4-RD(J)**4)
      RW(J)=0.78539816*DNST*(RW(J)**2-RD(J)**2)
      C2=1.5707963*SHM*(RS(J)**2-RD(J)**2)
      RS(J)=C1
      IF(C2)  202,202,201
  201 RD(J)=C1/C2
```

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```
          GO TO 203
    202  RD(J)=0.0
    203  CONTINUE
          READ (5,101) (LB(J),J=1,NB)
          WRITE(6,109)
          WRITE(6,101)(LB(J),J=1,NB)                                        85
          IF(NRP)  210,210,204                                              92
    204  WRITE(6,111)                                                       93
          WRITE(6,112)
          DO 205 J=1,NB                                                    101
          READ (5,106) PMX(J), PKX(J), POX(J), PMY(J), PKY(J), POY(J)      102
          WRITE(6,107)LB(J),PMX(J),PKX(J),POX(J),PMY(J),PKY(J),POY(J)
          PMX(J)=PMX(J)/386.069                                           105
    205  PMY(J)=PMY(J)/386.069                                            112
    210  IF(NPD)   214,217,211
    211  IF((NPD/2)*2-NPD) 213,212,212
    212  NPD1=NPD/2
          NPD2=-2
          GO TO 217
    213  NPD1=(NPD+1)/2
          NPD2=-1
          GO TO 217
    214  NPD1=-NPD
          IF((NPD1/2)*2-NPD1)  216,215,215
    215  NPD1=NPD1/2+1
          NPD2=0
          GO TO 217
    216  NPD1=(NPD1+1)/2
          NPD2=1
    217  NSP1=1
    230   READ (5,102) SPST, SPFN, SPIN, SFR, SCF
          READ (5,102) QZ, QZP                                            146
          WRITE(6,113)                                                    147
          WRITE(6,114)                                                    148
          WRITE(6,102)QZ,QZP                                              149
          WRITE(6,115)                                                    150
          DO 218 I=1,KQ2                                                  151
          READ (5,133) (WQ(I,J),J=1,KQ2)
          WRITE(6,133)(WQ(I,J),J=1,KQ2)                                   154
    218  CONTINUE                                                         159
          WRITE(6,126)
          DO 219 I=1,KQ2                                                  166
          READ (5,133) (WSQ(I,J),J=1,KQ2)
          WRITE(6,133)(WSQ(I,J),J=1,KQ2)                                  169
    219  CONTINUE                                                         174
          IF(NQ)  231,231,232
    231.  READ (5,102) QST, QFN, QINC
          GO TO 233
    232   READ (5,102) (QLST(J),J=1,NQ)                                   183
    233  SFR1=0.052359878*SFR
          WRITE(6,116)                                                    185
          IF(INC)  242,241,242                                            193
    241  K1=1
          GO TO 243
    242  K1=NH1
    243  DO 255 J=1,NB
```

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```
      WRITE(6,117)LP(J)
      IF(NPC) 257,244,250
  244 WRITE(6,118)                                                         201
      DO 245 I=1,K1
      K2=I-1                                                               204
      READ (5,106) BKXX(J,I), BCXX(J,I), BKXY(J,I), BCXY(J,I),
     1BKYX(J,I),   BCYX(J,I), BKYY(J,I), BCYY(J,I)                         I05
      WRITE(5,137)K2,BKXX(J,I),BCXX(J,I),BKXY(J,I),BCXY(J,I),BKYX(J,I),
     1BCYX(J,I),BKYY(J,I),BCYY(J,I)                                        207
  245 CONTINUE
      IF(INC)   255,246,255                                               216
  246 DO 247 I=2,NH1
      BKXX(J,I)=BKXX(J,1)
      BCXX(J,I)=BCXX(J,1)
      BKXY(J,I)=BKXY(J,1)
      BCXY(J,I)=BCXY(J,1)
      BKYX(J,I)=BKYX(J,1)
      BCYX(J,I)=BCYX(J,1)
      BKYY(J,I)=BKYY(J,1)
  247 BCYY(J,I)=BCYY(J,1)
      GO TO 255
  250 DO 254 K=1,NPC1
      WRITE(6,119)K
      WRITE(6,120)
      READ (5,102) PMIN(J,K), PADM(J,K),  PADK(J,K), PANG(J,K)            252
      WRITE(6,132)PMIN(J,K),PADM(J,K),PADK(J,K),PANG(J,K)                 253
      PMIN(J,K)=PMIN(J,K)/386.069                                         254
      PADM(J,K)=PADM(J,K)/386.069                                         259
      PANG(J,K)=0.3174532)3*PANG(J,K)
      WRITE(6,118)
      DO 251 I=1,K1
      K2=I-1                                                              270
      READ (5,106) PKXX(J,K,I), PCXX(J,K,I), PKXY(J,K,I),PCXY(J,K,I),
     1PKYX(J,K,I),   PCYX(J,K,I), PKYY(J,K,I),   PCYY(J,K,I)
      WRITE(6,107)K2,PKXX(J,K,I),PCXX(J,K,I),PKXY(J,K,I),PCXY(J,K,I),     273
     1PKYX(J,K,I),PCYX(J,K,I),PKYY(J,K,I),PCYY(J,K,I)
  251 CONTINUE                                                            282
      IF(INC) 254,252,254
  252 DO 253 I=2,NH1
      PKXX(J,K,I)=PKXX(J,K,1)
      PCXX(J,K,I)=PCXX(J,K,1)
      PKXY(J,K,I)=PKXY(J,K,1)
      PCXY(J,K,I)=PCXY(J,K,1)
      PKYX(J,K,I)=PKYX(J,K,1)
      PCYX(J,K,I)=PCYX(J,K,1)
      PKYY(J,K,I)=PKYY(J,K,1)
  253 PCYY(J,K,I)=PCYY(J,K,1)
  254 CONTINUE
  255 CONTINUE
      SPD=SPST
  260 WRITE(6,121)SPD
      SCF1=0.10471976*SPD
      SCF1=SCF*SCF1*SCF1                                                  320
      DO 519 IH=1,NH1
      NF=IH-1
      HN=NF
```

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```
      FRQ=HN*SPD*SFR1
      FQ2=FRQ*FRQ
      HN1=HN/2.0*SFR
      IF(NDIA) 401,402,401
  401 WRITE(6,122)NF,FRQ
      WRITE(6,123)
      WRITE(6,124)
  402 DO 425 J=1,NB
      IF(NPD)404,403,404
  403 D1=BKXX(J,IH)
      D2=BCXX(J,IH)*HN1
      D3=BKXY(J,IH)
      D4=BCXY(J,IH)*HN1
      D5=BKYX(J,IH)
      D6=BCYX(J,IH)*HN1
      D7=BKYY(J,IH)
      D8=BCYY(J,IH)*HN1
      GO TO 416
  404 D1=0.0
      D2=0.0
      D3=0.0
      D4=0.0
      D5=0.0
      D6=0.0
      D7=0.0
      D8=0.0
      DO 415 I=1,NPC1
      C1=FQ2*PMIN(J,I)
      A1=PKXX(J,I,IH)
      A2=PCXX(J,I,IH)*HN1
      A3=PKXY(J,I,IH)
      A4=PCXY(J,I,IH)*HN1
      A5=PKYX(J,I,IH)
      A6=PCYX(J,I,IH)*HN1
      A7=PKYY(J,I,IH)
      A8=PCYY(J,I,IH)*HN1
      C2=A7-C1
      C3=PADK(J,I)-FQ2*PADM(J,I)
      C4=A1+C3
      C5=C4*C2-A2*A8-A3*A5+A4*A6
      C6=C4*A8+C2*A2-A3*A6-A5*A4
      C7=C5*C5+C6*C6
      C11R=C3*(C5*C2+C6*A8)/C7
      C11E=C3*(C5*A8-C6*C2)/C7
      C12R=C1*(C5*A3+C6*A4)/C7
      C12E=C1*(C5*A4-C6*A3)/C7
      C21R=-C3*(C5*A5+C6*A6)/C7
      C21E=C3*(C6*A5-C5*A6)/C7
      C22R=-C1*(C5*C4+C6*A2)/C7
      C22E=C1*(C6*C4-C5*A2)/C7
      DKXX=C11R*A1-C11E*A2+C21R*A3-C21E*A4
      DCXX=C11R*A2+C11E*A1+C21R*A4+C21E*A3
      DKXY=C12R*A1-C12E*A2+C22R*A3-C22E*A4
      DCXY=C12R*A2+C12E*A1+C22R*A4+C22E*A3
      DKYX=C11R*A5-C11E*A6+C21R*A7-C21E*A8
      DCYX=C11R*A6+C11E*A5+C21R*A8+C21E*A7
```

326
327
328

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      DKYY=C12R*A5-C12E*A6+C22R*A7-C22E*A8
      DCYY=C12R*A6+C12E*A5+C22R*A8+C32E*A7
      C3=PANG(J,I)
      C1=COS(C3)
      C2=SIN(C3)
      C4=C1*C1
      C5=C2*C2
      A1=DKXX*C4+DKYY*C5
      A2=DCXX*C4+DCYY*C5
      A3=DKXY*C4-DKYX*C5
      A4=DCXY*C4-DCYX*C5
      A5=DKYX*C4-DKXY*C5
      A6=DCYX*C4-DCXY*C5
      A7=DKYY*C4+DKXX*C5
      A8=DCYY*C4+DCXX*C5
      IF(NPC2+1)  410,405,405
405   IF(I-1)  406,406,407
406   IF(NPD2)  409,409,410
407   IF(I-NPD1) 410,408,408
408   IF(NPC2)  410,409,409
409   D1=D1+A1
      D2=D2+A2
      D3=D3+A3
      D4=D4+A4
      D5=D5+A5
      D6=D6+A6
      D7=D7+A7
      D8=D8+A8
      GO TO 415
410   D1=D1+A1+A1
      D2=D2+A2+A2
      D3=D3+A3+A3
      D4=D4+A4+A4
      D5=D5+A5+A5
      D6=D6+A6+A6
      D7=D7+A7+A7
      D8=D8+A8+A8
415   CONTINUE
416   IF(NDIA)  417,420,
417   WRITE(5,107)LB        2,D3,D4,D5,D6,D7,D8
420   IF(NRP) 421,421,
421   SXX(J)=D1
      DXX(J)=D2
      SXY(J)=D3
      DXY(J)=D4
      SYX(J)=D5
      DYX(J)=D6
      SYY(J)=D7
      DYY(J)=D8
      GO TO 425
422   C1=PKX(J)-FQ2*PMX(J)
      C2=PKY(J)-FQ2*PMY(J)
      C3=FRQ*PDX(J)
      C4=FRQ*PDY(J)
      C5=D1+C1
      C6=D7+C2
```

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```
      C7=D2+C3
      C8=D8+C4
      A1=C5*C6-C7*C8-D3*D5+D4*D6
      A2=C5*C8+C6*C7-D3*D6-D4*D5
      C9=A1*A1+A2*A2
      A3=C1*C6-C3*C8
      A4=C1*C8+C3*C6
      C11R=(A3*A1+A4*A2)/C9
      C11E=(A4*A1-A3*A2)/C9
      A3=C2*D3-C4*D4
      A4=C2*D4+C4*D3
      C12R=-(A3*A1+A4*A2)/C9
      C12E=-(A4*A1-A3*A2)/C9
      A3=C1*D5-C3*D6
      A4=C1*D6+C3*D5
      C21R=-(A3*A1+A4*A2)/C9
      C21E=-(A4*A1-A3*A2)/C9
      A3=C2*C5-C4*C7
      A4=C2*C7+C4*C5
      C22R=(A3*A1+A4*A2)/C9
      C22E=(A4*A1-A3*A2)/C9
      SXX(J)=C11R*D1-C11E*D2+C21R*D3-C21E*D4
      DXX(J)=C11R*D2+C11E*D1+C21R*D4+C21E*D3
      SXY(J)=C12R*D1-C12E*D2+C22R*D3-C22E*D4
      DXY(J)=C12R*D2+C12E*D1+C22R*D4+C22E*D3
      SYX(J)=C11R*D5-C11E*D6+C21R*D7-C21E*D8
      DYX(J)=C11R*D6+C11E*D5+C21R*D8+C21E*D7
      SYY(J)=C12R*D5-C12E*D6+C22R*D7-C22E*D8
      DYY(J)=C12R*D6+C12E*D5+C22R*D8+C22E*D7
  425 CONTINUE
      DO 449 J=1,NS1
      C1=RS(J)
      C2=FQ2*RW(J)
      C3=RD(J)
      C4=C2/C1
      C5=SQRT(C4)
      C6=SQRT(C5)
      C7=RL(J)
      IF(C6*C7-0.03)  441,441,442
  441 C8=C2*C7
      B1(J)=1.0
      B2(J)=1.0
      B3(J)=C7
      B6(J)=C7/C1
      B4(J)=B6(J)/2.0*C7
      B7(J)=B4(J)/3.0*C7-C3*C7/C1*2.0
      B5(J)=C2*B7(J)
      B8(J)=C8
      B9(J)=C8/2.0*C7
      B10(J)=B9(J)/3.0*C7
      GO TO 449
  442 C8=C3*C3*C4
      C9=C3*C5
      IF(C8-0.0002)  443,443,444
  443 C8=1.0+0.5*C8
      GO TO 445
```

     412
     413

```
444 CB=SQRT(1.0+C8)
443 A5=C5*(C8-C9)                                                   437
    A6=C5*(C9+C9)
    A9=A5+A6
    A3=SQRT(A5)
    A4=SQRT(A6)                                                     439
    A7=A3*A5                                                        440
    A8=A4*A6
    A1=A3*C7
    A2=A4*C7
    D1=COS(A2)/A9                                                   441
    D2=SIN(A2)/A9                                                   442
    D5=EXP(A1)                                                      443
    D4=1.0/D5
    D3=0.5*(D5+D4)/A9
    D4=0.5*(D5-D4)/A9
    B1(J)=A5*D3+A6*D1
    B2(J)=A6*D3+A5*D1
    B3(J)=A3*D4+A4*D2
    B8(J)=C2*B3(J)
    B4(J)=(D3-D1)/C1
    B9(J)=C2*(D3-C1)
    B5(J)=C5*(A4*C4-A3*D2)
    B10(J)=C1*B5(J)
    C8=C1*C5
    B6(J)=(A8*D4+A7*C2)/C8
    B7(J)=(A7*D4-A8*D2)/C2
449 CONTINUE
    DO 455 J=1,NS
    C1=FQ2*RM(J)
    DVXA(J)=C1
    DVYA(J)=C1
    DVXB(J)=0.0
    DVXC(J)=0.0
    DVXD(J)=0.0
    DVYB(J)=0.0
    DVYC(J)=0.0
    DVYD(J)=0.0
    DMXA(J)=FQ2*RIT(J)
    DVUX(J)=0.0
    DVUY(J)=0.0
    DMUX(J)=0.0
455 DMUY(J)=0.0
    DO 456 J=1,NB
    K1=LB(J)
    DVXA(K1)=DVXA(K1)-SXX(J)
    DVXB(K1)=DXX(J)
    DVXC(K1)=SXY(J)
    DVXD(K1)=DXY(J)
    DVYA(K1)=DVYA(K1)-SYY(J)
    DVYB(K1)=DYY(J)
    DVYC(K1)=SYX(J)
456 DVYD(K1)=DYX(J)
    DVXA(KB)=DVXA(KB)+QZ
    DVYA(KB)=DVYA(KB)+QZ
    DMXA(KB)=DMXA(KB)+QZP
```

```
      DO 485 I=L,KQ3
      BMXC=0.0
      BMXS=0.0
      BMYC=0.0
      BMYS=0.0
      VXC=0.0
      VXS=0.0
      VYC=0.0
      VYS=0.0
      XC=0.0
      XS=0.0
      YC=0.0
      YS=0.0
      DXC=0.0
      DXS=0.0
      DYC=0.0
      DYS=0.0
      DVUX(K8)=0.0
      DVUY(K8)=0.0
      DMUX(K8)=0.0
      DMUY(K8)=0.0
      GO TO(461,462,463,464,465,469,468,472),I
  461 XC=0.001
      GO TO 475
  462 YC=0.001
      GO TO 475
  463 DXC=0.001
      GO TO 475
  464 DYC=0.001
      GO TO 475
  465 IF(KC) 467,466,466
  466 IF(KA) 468,467,467
  467 DVUX(K8)=1.0
      GO TO 475
  468 DMUX(K8)=1.0
      GO TO 475
  469 IF(KC) 471,470,470
  470 IF(KA) 472,471,471
  471 DVUY(K8)=1.0
      GO TO 475
  472 DMUY(K8)=1.0
  475 DO 480 J=1,NS
      C1=DMXA(J)
      C2=FRQ*SPD*RIP(J)
      IF(J-K8) 477,476,477
  476 CMR(1,I)=XC
      CME(1,I)=XS
      CMR(2,I)=YC
      CME(2,I)=YS
      CMR(3,I)=DXC
      CME(3,I)=DXS
      CMR(4,I)=DYC
      CME(4,I)=DYS
  477 A1=BMXC-C1*DXC-C2*DYS-DMUX(J)
      A2=BMXS-C1*DXS+C2*DYC
      A3=BMYC-C1*DYC+C2*DXS-DMUY(J)
```

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```
      A4=BMYS-C1*DYS-C2*DXC
      A5=VXC+DVXA(J)*XC+DVXB(J)*XS-DVXC(J)*YC+DVXD(J)*YS+DVUX(J)
      A6=VXS-DVXB(J)*XC+DVXA(J)*XS-DVXD(J)*YC-DVXC(J)*YS
      A7=VYC-DVYC(J)*XC+DVYD(J)*XS+DVYA(J)*YC+DVYB(J)*YS+DVUY(J)
      A8=VYS-DVYD(J)*XC-DVYC(J)*XS-DVYB(J)*YC+DVYA(J)*YS
      IF(NS-J)  46J,45C,478
  478 C1=XC
      C2=XS
      C3=YC
      C4=YS
      BMXC=C1*B9(J)+DXC*B10(J)+A1*B2(J)+A5*B3(J)
      BMXS=C2*B9(J)+DXS*B10(J)+A2*B2(J)+A6*B3(J)
      BMYC=C3*B9(J)+DYC*B10(J)+A3*B2(J)+A7*B3(J)
      BMYS=C4*B9(J)+DYS*B10(J)+A4*B2(J)+A8*B3(J)
      VXC=C1*B8(J)+CXC*B9(J)+A1*B5(J)+A5*B1(J)
      VXS=C2*B8(J)+CXS*P9(J)+A2*B5(J)+A6*B1(J)
      VYC=C3*B8(J)+CYC*B9(J)+A3*B5(J)+A7*B1(J)
      VYS=C4*B9(J)+CYS*B9(J)+A4*B5(J)+A8*B1(J)
      XC=C1*B1(J)+DXC*B3(J)+A1*B4(J)+A5*B7(J)
      XS=C2*B1(J)+DXS*B3(J)+A2*B4(J)+A6*B7(J)
      YC=C3*B1(J)+DYC*B3(J)+A3*B4(J)+A7*B7(J)
      YS=C4*B1(J)+DYS*B3(J)+A4*B4(J)+A9*B7(J)
      DXC=C1*B5(J)+CXC*B2(J)+A1*B6(J)+A5*B4(J)
      DXS=C2*B5(J)+CXS*B2(J)+A2*B6(J)+A6*B4(J)
      DYC=C3*B5(J)+CYC*B2(J)+A3*B6(J)+A7*B4(J)
      DYS=C4*B5(J)+CYS*B2(J)+A4*B6(J)+A9*B4(J)
  480 CONTINUE
      AMR(1,I)=A1
      AME(1,I)=A2
      AMR(2,I)=A3
      AME(2,I)=A4
      AMR(3,I)=A5
      AME(3,I)=A6
      AMR(4,I)=A7
      AME(4,I)=A8
  485 CONTINUE
      DO 486 J=1,4
      DO 486 I=1,4
      WR(I,J)=AMR(I,J)
  486 WE(I,J)=AME(I,J)
      CALL MATINV(WR,4,WCC,0,DVN,IC)                               666
      GO TO(481,460),IC
  460 WRITE(6,130)                                                 668
      WRITE(6,131)NF,FRQ                                          669
      GO TO 519
  481 IF(NF) 457,457,482
  457 DO 459 I=1,4
      DO 458 J=1,4
      AME(I,J)=0.0
      UR(I,J)=WR(I,J)
  458 UE(I,J)=0.0
      GO TO 459
  482 CALL MATINV(WE,4,WCC,0,DED,ID)                               687
      GO TO(483,457),IC
  483 DO 488 I=1,4
      DO 488 J=1,4
```

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```
      C1=0.0
      DO 487 K=1,4
  487 C1=C1+WR(I,K)*AME(K,J)+WE(I,K)*AMR(K,J)
  488 WA(I,J)=C1
      CALL MATINV(WA,4,WCC,0,DVN,ID)                                                                  706
      GO TO (702,701),ID
  701 WRITE(6,137)                                                                                    708
      WRITE(6,131)NF,FRQ                                                                              709
      GO TO 519
  702 DO 490 I=1,4
      DO 490 J=1,4
      C1=0.0
      C2=0.0
      DO 489 K=1,4
      C1=C1+WA(I,K)*WE(K,J)
  489 C2=C2-WA(I,K)*WR(K,J)
      UR(I,J)=C1
  490 UE(I,J)=C2
  459 DO 492 I=1,4
      DO 492 J=1,4
      C1=0.0
      C2=0.0
      DO 491 K=1,4
      C1=C1+CMR(I,K)*UR(K,J)-CME(I,K)*UE(K,J)
  491 C2=C2+CMR(I,K)*UE(K,J)+CME(I,K)*UR(K,J)
      WR(I,J)=C1
  492 WE(I,J)=C2
      DO 494 I=1,4
      DO 494 J=1,KQ2
      C1=CMR(I,J+4)
      C2=CME(I,J+4)
      DO 493 K=1,4
      C1=C1-WR(I,K)*AMR(K,J+4)+WE(I,K)*AME(K,J+4)
  493 C2=C2-WR(I,K)*AME(K,J+4)-WE(I,K)*AMR(K,J+4)
      UR(I,J)=C1
  494 UE(I,J)=C2
      IF(KC) 500,499,499
  499 DO 497 I=1,2
      DO 497 J=1,2
      IF(KC) 500,495,496
  495 EMR(I,J)=UR(I,J)
      EME(I,J)=UE(I,J)
      GO TO 497
  496 EMR(I,J)=UR(I+2,J)
      EME(I,J)=UE(I+2,J)
  497 CONTINUE
      C1=EMR(1,1)*EMR(2,2)-EMR(1,2)*EMR(2,1)-EME(1,1)*EME(2,2)+
     1EME(1,2)*EME(2,1)
      C2=EMR(1,1)*EME(2,2)+EMR(2,2)*EME(1,1)-EMR(1,2)*EME(2,1)-
     1EMR(2,1)*EME(1,2)
      C3=(C1*C1+C2*C2)*SCFI
      C1=C1/C3
      C2=C2/C3
      GR(1,1,IH)=C1*EMR(2,2)+C2*EME(2,2)
      GR(1,2,IH)=-C1*EMR(1,2)-C2*EME(1,2)
      GR(2,1,IH)=-C1*EMR(2,1)-C2*EME(2,1)
```

```
      GR(2,2,IH)= C1*EPR(1,1)+C2*EME(1,1)
      GE(1,1,IH)= C1*EPE(2,2)-C2*EMR(2,2)
      GE(1,2,IH)=-C1*EPE(1,2)+C2*EMR(1,2)
      GE(2,1,IH)=-C1*EPE(2,1)+C2*EMR(2,1)
      GE(2,2,IH)= C1*EPE(1,1)-C2*EMR(1,1)
      GO TO 518
  500 DO 501 I=1,4
      DO 501 J=1,4
      WR(I,J)=UR(I,J)
  501 WE(I,J)=UE(I,J)
      CALL MATINV(WR,4,WCC,0,DVN,ID)                                816
      GO TO (503,502),IC
  502 WRITE(6,132)                                                  818
      WRITE(6,131)NF,FRQ                                            819
      GO TO 519
  503 IF(NF) 504,504,506
  504 DO 505 I=1,4
      DO 505 J=1,4
      GE(I,J,IH)=0.0
  505 GR(I,J,IH)=WR(I,J)/SCF1
      GO TO 518
  506 CALL MATINV(WE,4,WCC,0,DED,ID)                                836
      GO TO (507,504),ID
  507 DO 509 I=1,4
      DO 509 J=1,4
      C1=0.0
      DO 508 K=1,4
  508 C1=C1+WR(I,K)*UE(K,J)+WE(I,K)*UR(K,J)
  509 WA(I,J)=C1
      CALL MATINV(WA,4,WCC,0,DVN,ID)                                855
      GO TO (704,703),ID
  703 WRITE(6,138)                                                  857
      WRITE(6,131)NF,FRQ                                            858
      GO TO 519
  704 DO 517 I=1,4
      DO 517  J=1,4
      C1=0.0
      C2=0.0
      DO 516 K=1,4
      C1=C1+WA(I,K)*WE(K,J)
  516 C2=C2-WA(I,K)*WR(K,J)
      GR(I,J,IH)=C1/SCF1
  517 GE(I,J,IH)=C2/SCF1
  518 IF(NDIA) 498,519,498
  498 WRITE(6,125)                                                  880
      WRITE(6,127)                                                  881
      WRITE(6,133)((GR(I,J,IH),J=1,4),I=1,4)                        882
      WRITE(6,128)                                                  890
      WRITE(6,133)((GE(I,J,IH),J=1,4),I=1,4)                        891
  519 CONTINUE
      K3=0
      K4=0
      K5=0
      K6=0
      K7=1
      WRITE(6,129)NCIA                                              906
```

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```
      QX1=QST-QINC
      NQ1=1
510   IF(NQ) 511,511,512
511   QX1=QX1+QINC
      IF(QFN+0.000001-QX1)  599,599,515
512   IF(NQ-NQ1)  599,513,513
513   QX1=QLST(NQ1)
      NQ1=NQ1+1
515   QX2=0.5*QX1/SCF1
      DO 521 I=1,KQ2
      DO 521 J=1,KQ2
      WA(I,J)=QX2*WQ(I,J)
      WB(I,J)=-QX2*WSQ(I,J)
      AMR(I,J)=0.0
      AME(I,J)=0.0
      CMR(I,J)=0.0
521   CME(I,J)=0.0
      DO 553 IH=2,NH1
      K1=NH1+2-IH
      IF((K1/2)*2-K1)  523,525,525
523   K2=0
      DO 524 I=1,KQ2
      DO 524 J=1,KQ2
      UR(I,J)=AMR(I,J)
524   UE(I,J)=AME(I,J)
      GO TO 527
525   K2=1
      DO 526 I=1,KQ2
      DO 526 J=1,KQ2
      UR(I,J)=CMR(I,J)
526   UE(I,J)=CME(I,J)
527   IF(NDIA)  705,706,706
705   WRITE(6,135)K1                                          967
      WRITE(6,133)((UR(I,J),J=1,KQ2),I=1,KQ2)                 968
      WRITE(6,133)((UE(I,J),J=1,KQ2),I=1,KQ2)                 976
706   DO 529 I=1,KQ2
      DO 529 J=1,KQ2
      C1=GR(I,J,K1)
      C2=GE(I,J,K1)
      DO 528 K=1,KQ2
      C1=C1+WA(I,K)*UR(K,J)-WB(I,K)*UE(K,J)
528   C2=C2+WA(I,K)*UE(K,J)+WB(I,K)*UR(K,J)
      WR(I,J)=C1
529   WE(I,J)=C2
      IF(K1-3) 550,522,522
522   IF(KC) 535,530,530
530   C1=WR(1,1)*WR(2,2)-WR(1,2)*WR(2,1)-WE(1,1)*WE(2,2)+WE(1,2)*WE(2,1)
      C2=WR(1,1)*WE(2,2)+WR(2,2)*WE(1,1)-WR(1,2)*WE(2,1)-WR(2,1)*WE(1,2)
      C3=C1*C1+C2*C2
      C1=-C1/C3
      C2=-C2/C3
      EMR(1,1)=C1*WR(2,2)+C2*WE(2,2)
      EME(1,1)=C1*WE(2,2)-C2*WR(2,2)
      EMR(1,2)=-C1*WR(1,2)-C2*WE(1,2)
      EME(1,2)=C2*WR(1,2)-C1*WE(1,2)
      EMR(2,1)=-C1*WR(2,1)-C2*WE(2,1)
```

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```
      EME(2,1)=C2*WR(2,1)-C1*WE(2,1)
      EMR(2,2)=C1*WR(1,1)+C2*WE(1,1)
      EME(2,2)=C1*WE(1,1)-C2*WR(1,1)
      GO TO 546
  535 DO 536 I=1,4
      DO 536 J=1,4
      UR(I,J)=WR(I,J)
  536 UE(I,J)=WE(I,J)
      CALL MATINV(UR,4,WCC,0,DVN,ID)
      GO TO (538,537),ID
  537 WRITE(6,134)K1                                                1025
      GO TO 553
  538 CALL MATINV(UE,4,WCC,0,DED,ID)                                1027
      GO TO (541,539),ID
  539 DO 540 I=1,4                                                  1030
      DO 540 J=1,4
      EMR(I,J)=-UR(I,J)
  540 EME(I,J)=0.0
      GO TO 546
  541 DO 543 I=1,4
      DO 543 J=1,4
      C1=0.0
      DO 542 K=1,4
  542 C1=C1+UR(I,K)*WE(K,J)+UE(I,K)*WR(K,J)
  543 AMR(I,J)=C1
      CALL MATINV(AMR,4,WCC,0,DVN,ID)
      GO TO (708,707),IC
  707 WRITE(6,139)K1                                                1061
      GO TO 553
  708 DO 545 I=1,4                                                  1063
      DO 545 J=1,4
      C1=0.0
      C2=0.0
      DO 544 K=1,4
      C1=C1+AMR(I,K)*UE(K,J)
  544 C2=C2-AMR(I,K)*UR(K,J)
      EMR(I,J)=-C1
  545 EME(I,J)=-C2
  546 DO 534 I=1,KQ2
      DO 534 J=1,KQ2
      C1=0.0
      C2=0.0
      DO 531 K=1,KQ2
      C1=C1+EMR(I,K)*WA(K,J)+EME(I,K)*WB(K,J)
  531 C2=C2+EME(I,K)*WA(K,J)-EMR(I,K)*WB(K,J)
      IF(K2)   533,532,533
  532 AMR(I,J)=C1
      AME(I,J)=C2
      GO TO 534
  533 CMR(I,J)=C1
      CME(I,J)=C2
  534 CONTINUE
      IF(K1-3) 550,547,553
  547 DO 549 I=1,KQ2
      DO 549 J=1,KQ2
      C1=GR(I,J,1)
```

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```
      DO 548 K=1,KQ2
548 C1=C1+2.0*WA(I,K)*AMR(K,J)-2.0*WB(I,K)*AME(K,J)
549 DEVN(I,J)=C1
    GO TO 553
550 K8=-1
    DO 552 I=1,KQ2
    K8=K8+2
    K9=-1
    DO 551 J=1,KQ2
    K9=K9+2
    DEDD(K8,K9)=WR(I,J)+WA(I,J)
    DEDD(K8,K9+1)=-WE(I,J)-WB(I,J)
    DEDD(K8+1,K9)=WE(I,J)-WB(I,J)
551 DEDD(K8+1,K9+1)=WR(I,J)-WA(I,J)
552 CONTINUE
553 CONTINUE
    IF(NDIA)   709,710,710
709 WRITE(6,140)                                                             1160
    WRITE(6,133)((DEVN(I,J),J=1,KQ2),I=1,KQ2)                                1161
    WRITE(6,141)                                                             1171
    WRITE(6,133)((DEDD(I,J),J=1,KQ1),I=1,KQ1)                                1172
710 CALL MATINV(DEVN,KQ2,WCC,0,DVN,ID)                                       1183
    GO TO (582,581),ID
581 DVN=0.0
    IJKL=1
    WRITE(6,136) IJKL                                                        1186
582 CALL MATINV(DEDD,KQ1,WCC,0,DED,IC)                                       1188
    GO TO (584,583),ID
583 DED=0.0
    IJKL=2
    WRITE(6,136) IJKL                                                        1191
584 WRITE(6,102)QX1,DVN,DED                                                  1192
    IF(K7)   554,555,554
554 K7=0
    DVN1=DVN
    GO TO 561
555 IF(K6)   556,556,559
556 IF(K3) 565,557,564
557 IF(DVN*DVN1)   562,558,558
558 DVN1=DVN
559 IF(K4)   571,560,570
560 IF(DED*DED1)   566,561,561
561 DED1=DED
    Q1=QX1
    K5=0
    GO TO 510
562 K3=1
    DVN3=DVN
    DED3=DED
    D3=DVN
563 Q3=QX1
    QX1=0.5*(Q1+Q3)
    GO TO 515
564 K3=-1
    D1=DVN1
    D2=DVN
```

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MTI-3924

```
        DED2=DED
        Q2=QX1
        GO TO 575
  565 K3=0
      K5=1
      QX1=Q3
      DVN=DVN3
      DED=DED3
      GO TO 558
  566 K6=1
      IF(K5)  569,569,567
  567 K5=0
  568 K4=-1
      D1=DED1
      D2=DED2
      D3=DED3
      GO TO 575
  569 K4=1
      DED3=CED
      GO TO 563
  570 Q2=QX1
      DED2=DED
      GO TO 568
  571 K4=0
      K6=0
      QX1=Q3
      DVN=DVN1
      DED=DED3
      GO TO 561
  575 C1=Q2-Q1
      C2=Q3-Q2
      C3=Q3-Q1
      C4=C2*C3
      C3=C1*C3
      C3=(D1-D2)/C3
      C4=(D3-D2)/C4
      C2=C1*C4-C2*C3
      C1=C4+C3
      IF(C1)  577,576,577
  576 C5=-D2/C2
      GO TO 580
  577 C3=0.5*C2/C1
      C4=SQRT(C3*C3-D2/C1)
      IF(C3)   579,578,578
  578 C5=C4-C3
      GO TO 580
  579 C5=-C4-C3
  580 QX1=Q2+C5
      GO TO 515
  599 SPD=SPD+SPIN
      IF(SPFN+0.000001-SPD)  600,600,260
  600 NSP1=NSP1+1
      IF(NSP-NSP1)  601,230,230
  601 IF(INP)  602,190,602
  602 STOP
  99  FORMAT (1H1)
```

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KRI-3929

```
100 FORMAT(72H
   1
101 FORMAT(12I5)
102 FORMAT(1P5E14.6)
103 FORMAT(118HOSTATIONS  BEARINGS    MAGN.ST   F+M/F/M   RIG.PED   NO.
   1PADS  COMPRESS  HARMONICS    NO.Q     NO.SPEEDS  DIAGNOS     INPUT)
104 FORMAT(I6,11I10)
105 FORMAT(42HO YOUNGS MOD.      DENSITY    (SHAPE FACT)+G)
106 FORMAT(8E9.2)
107 FORMAT(I5,1PE16.6,1P7E14.6)
108 FORMAT(104H STATION    MASS,LBS    POLAR MOM.IN. TRANSV.MOM.IN      L
   1ENGTH   OUT.DIA(STIFF) OUT.DIA(MASS)   INNER DIA.)
109 FORMAT(17HOBEARING STATIONS)
110 FORMAT(11HOROTOR DATA)
111 FORMAT(14HOPEDESTAL DATA)
112 FORMAT(89H STATION  MASS-X,LBS    STIFFNESS-X    DAMPING-X      MASS
   1-Y,LBS    STIFFNESS-Y    DAMPING-Y)
113 FORMAT(20H1MAGNETIC FORCE DATA)
114 FORMAT(27HO  Q(0),FORCE    Q(0),MOMENT)
115 FORMAT(28HOMATRIX OF COSINE COMPONENTS)
116 FORMAT(//13HOBEARING DATA)
117 FORMAT(19HOBEARING AT STATION,I3)
118 FORMAT(9HOHARMONIC4X3HFXX10X5HW+BXX10X3HKXY10X5HW+BXY10X3HKYX10X5H
   1W+BYX10X3HKYY10X5HW+BYY)
119 FORMAT(8HOPAD NO.,I3)
120 FORMAT(55H PITCH MOM.IN.    PAD MASS   PIVOT STIFFN.  PIVOT ANGLE)
121 FORMAT(13H1ROTOR SPEED=,1PE13.6,4H RPM)
122 FORMAT(//13HOHARMONIC NO.,I3,11H,FREQUENCY=,1PE13.6,8H RAD/SEC)
123 FORMAT(21HOBEARING COEFFICIENTS)
124 FORMAT(8H STATION5X3HKXX9X7HFRQ+BXX9X3HKXY9X7HFRQ+BXY9X3HKYX9X7HFR
   1Q+BYX9X3HKYY9X7HFRQ+BYY)
125 FORMAT(31HOROTOR-BEARING IMPEDANCE MATRIX)
126 FORMAT(26HOMATRIX OF SINE COMPONENTS)
127 FORMAT(10HOREAL PART)
128 FORMAT(15HOIMAGINARY PART)
129 FORMAT(11,9X3HOXX7X25HEVEN DETERM.   ODD DETERM.)
130 FORMAT(34HOREAL PART OF A-MATRIX IS SINGULAR)
131 FORMAT(10H HARMONIC=,I3,12H  FREQUENCY=,1PE13.6)
132 FORMAT(42HOREAL PART OF IMPEDANCE MATRIX IS SINGULAR)
133 FORMAT(1P4E14.6)
134 FORMAT(37HOREAL PART OF S-MATRIX IS SINGULAR,K=,I3)
135 FORMAT(18HOS-MATRIX FOR K+1=,I3)
136 FORMAT(20HODETERMINANT ZERO AT,I3)
137 FORMAT(35HGINVERSION MATRIX FOR A IS SINGULAR)
138 FORMAT(35HOINVERSION MATRIX FOR E IS SINGULAR)
139 FORMAT(38HOINVERSION MATRIX FOR S IS SINGULAR,K=I3)
140 FORMAT(17HOEVEN DETERMINANT)
141 FORMAT(16HOODD DETERMINANT)
    END
```

```
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS    MAT1   2
C     NOVEMBER 1692   S GOOD   DAVID TAYLOR MODEL BASIN   AM MAT1         MAT1   3
C                                                                        MAT1   4
      SUBROUTINE MATINV(A,N1,B,M1,DETERM,ID)                             MAT1   5
C                                                                        MAT1   6
C     GENERAL FORM OF DIMENSION STATEMENT                                MAT1   7
C                                                                        MAT1   8
      DIMENSION A(8,8),B(8,1)
      DIMENSION INDEX(8,3)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)          MAT1  11
C                                                                        MAT1  12
C     INITIALIZATION                                                     MAT1  13
C                                                                        MAT1  14
      M=M1                                                               MAT1  15
      N=N1                                                               MAT1  16
      DO 8 I=1,N
      K1=1
      K2=1
      DO 6 J=1,N
      IF(A(I,J)) 3,4,3
    3 K1=0
    4 IF(A(J,I)) 5,6,5
    5 K2=0
    6 CONTINUE
      IF(K1+K2) 8,8,7
    7 ID=2
      DETERM=0.0
      GO TO 740
    8 CONTINUE
   10 DETERM=1.0
   15 DO 20 J=1,N                                                        MAT1  18
   20 INDEX(J,3) = 0                                                     MAT1  19
   30 DO 550 I=1,N                                                       MAT1  20
C                                                                        MAT1  21
C     SEARCH FOR PIVOT ELEMENT                                           MAT1  22
C                                                                        MAT1  23
   40 AMAX=0.0                                                           MAT1  24
   45 DO 105 J=1,N                                                       MAT1  25
      IF(INDEX(J,3)-1) 60, 105, 60                                       MAT1  26
   60 DO 100 K=1,N                                                       MAT1  27
      IF(INDEX(K,3)-1) 80, 100, 715                                      MAT1  28
   80 IF(AMAX-ABS(A(J,K))) 85,100,100                                    MAT1  29
   85 IROW=J                                                             MAT1  30
   90 ICOLUM=K                                                           MAT1  31
      AMAX=ABS(A(J,K))                                                   MAT1  32
  100 CONTINUE                                                           MAT1  33
  105 CONTINUE                                                           MAT1  34
      INDEX(ICOLUM,3) = INDEX(ICOLUM,3) +1                               MAT1  35
  260 INDEX(I,1)=IROW                                                    MAT1  36
  270 INDEX(I,2)=ICOLUM                                                  MAT1  37
C                                                                        MAT1  38
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL                  MAT1  39
C                                                                        MAT1  40
  130 IF (IROW-ICOLUM) 140, 310, 140                                     MAT1  41
  140 DETERM=-DETERM                                                     MAT1  42
```

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```
 150 DO 200 L=1,N                                              MAT1  43
 160 SWAP=A(IROW,L)                                            MAT1  44
 170 A(IROW,L)=A(ICOLUM,L)                                     MAT1  45
 200 A(ICOLUM,L)=SWAP                                          MAT1  46
     IF(M) 310, 310, 210                                       MAT1  47
 210 DO 250 L=1, M                                             MAT1  48
 220 SWAP=B(IROW,L)                                            MAT1  49
 230 B(IROW,L)=B(ICOLUM,L)                                     MAT1  50
 250 B(ICOLUM,L)=SWAP                                          MAT1  51
C                                                              MAT1  52
C    DIVIDE PIVOT ROW BY PIVOT ELEMENT                         MAT1  53
C                                                              MAT1  54
 310 PIVOT   =A(ICOLUM,ICOLUM)                                 MAT1  55
     DETERM=DETERM*PIVOT                                       MAT1  56
 330 A(ICOLUM,ICOLUM)=1.0                                      MAT1  57
 340 DO 350 L=1,N                                              MAT1  58
 350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT                             MAT1  59
 355 IF(M) 380, 380, 360                                       MAT1  60
 360 DO 370 L=1,M                                              MAT1  61
 370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT                             MAT1  62
C                                                              MAT1  63
C    REDUCE NON-PIVOT ROWS                                     MAT1  64
C                                                              MAT1  65
 380 DO 550 L1=1,N                                             MAT1  66
 390 IF(L1-ICOLUM) 400, 550, 400                               MAT1  67
 400 T=A(L1,ICOLUM)                                            MAT1  68
 420 A(L1,ICOLUM)=0.0                                          MAT1  69
 430 DO 450 L=1,N                                              MAT1  70
 450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T                             MAT1  71
 455 IF(M) 550, 550, 460                                       MAT1  72
 460 DO 500 L=1,M                                              MAT1  73
 500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T                             MAT1  74
 550 CONTINUE                                                  MAT1  75
C                                                              MAT1  76
C    INTERCHANGE COLUMNS                                       MAT1  77
C                                                              MAT1  78
 600 DO 710 I=1,N                                              MAT1  79
 610 L=N+1-I                                                   MAT1  80
 620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630                  MAT1  81
 630 JROW=INDEX(L,1)                                           MAT1  82
 640 JCOLUM=INDEX(L,2)                                         MAT1  83
 650 DO 705 K=1,N                                              MAT1  84
 660 SWAP=A(K,JROW)                                            MAT1  85
 670 A(K,JROW)=A(K,JCOLUM)                                     MAT1  86
 700 A(K,JCOLUM)=SWAP                                          MAT1  87
 705 CONTINUE                                                  MAT1  88
 710 CONTINUE                                                  MAT1  89
     DO 730 K = 1,N                                            MAT1  90
     IF(INDEX(K,3) -1) 715,720,715                            MAT1  91
 715 ID =2                                                     MAT1  92
     GO TO 740                                                 MAT1  93
 720 CONTINUE                                                  MAT1  94
 730 CONTINUE                                                  MAT1  95
     ID=1                                                      MAT1  96
 740 RETURN                                                    MAT1  97
     END                                                       MAT1  99
```

NTI-3936

```
TURBINE DRIVEN 4 POLE ALTERNATOR,STABILITY CHECK    4-11-1967
   11      2      7     -1      0      0      1     10      0      1      0      1
  30000000.0      0.283        8630000.0
   6.15    62.4      31.0      1.3     3.5      3.5      3.339
   1.61     0.0       0.0      1.45    3.5      3.6      3.2
   0.0      0.0       0.0      2.37    3.5      3.65     3.062
   0.0      0.0       0.0      4.83    3.5      3.65     3.062
   0.0      0.0       0.0      1.67    2.775    3.25     2.0
   3.1     12.0      12.0      2.1     3.5      3.7      1.04
   0.0      0.0       0.0      2.1     3.5      3.7      1.04
   3.1     12.0      12.0      1.67    2.775    3.25     2.0
   0.0      0.0       0.0      3.49    3.5      3.65     3.062
   0.0      0.0       0.0      4.39    3.5      3.65     3.062
   9.21    56.0      28.0      0.0     1.0      0.0      0.0
     4    10
  10300.0      20300.0       1000.0        2.0           1.0
  20000.0      88200.0
   0.0      0.0       4.2      0.0
   0.0      0.0       0.0     -4.2
   4.2      0.0       0.0      0.0
   0.0     -4.2       0.0      0.0
   0.0      0.0       0.0     -4.2
   0.0      0.0      -4.2      0.0
   0.0     -4.2       0.0      0.0
  -4.2      0.0       0.0      0.0
   0.0   1000000.0   10000.0
  111915.8    0.0       0.0      0.0      0.0      0.0    111915.8    0.0
  196262.3    0.0       0.0      0.0      0.0      0.0    196262.3    0.0
  197298.6    0.0       0.0      0.0      0.0      0.0    197298.6    0.0
  184631.7    0.0       0.0      0.0      0.0      0.0    184631.7    0.0
  168424.7    0.0       0.0      0.0      0.0      0.0    168424.7    0.0
  126272.4    0.0       0.0      0.0      0.0      0.0    126272.4    0.0
  254752.4    0.0       0.0      0.0      0.0      0.0    254752.4    0.0
  217609.4    0.0       0.0      0.0      0.0      0.0    217609.4    0.0
  181060.1    0.0       0.0      0.0      0.0      0.0    181060.1    0.0
  121742.5    0.0       0.0      0.0      0.0      0.0    121742.5    0.0
 -349724.4    0.0       0.0      0.0      0.0      0.0   -349724.4    0.0
  111915.8    0.0       0.0      0.0      0.0      0.0    111915.8    0.0
  196262.3    0.0       0.0      0.0      0.0      0.0    196262.3    0.0
  197298.6    0.0       0.0      0.0      0.0      0.0    197298.6    0.0
  184631.7    0.0       0.0      0.0      0.0      0.0    184631.7    0.0
  168424.7    0.0       0.0      0.0      0.0      0.0    168424.7    0.0
  126272.4    0.0       0.0      0.0      0.0      0.0    126272.4    0.0
  254752.4    0.0       0.0      0.0      0.0      0.0    254752.4    0.0
  217609.4    0.0       0.0      0.0      0.0      0.0    217609.4    0.0
  181060.1    0.0       0.0      0.0      0.0      0.0    181060.1    0.0
  121742.5    0.0       0.0      0.0      0.0      0.0    121742.5    0.0
 -349724.4    0.0       0.0      0.0      0.0      0.0   -349724.4    0.0
```

TURBINE DRIVEN 4 POLE ALTERNATOR, STABILITY CHECK 4-11-1967

STATIONS	BEARINGS	MAGN.ST	F+M/F/M	RIG.PED	NO.PADS	COMPRESS	HARMONICS	NO.Q	NO.SPEEDS	DIAGNOS	INPUT
11	2	7	-1	0	0	1	10	0	1	0	1

YOUNGS MOD.   DENSITY   (SHAPE FACT)*G
3.030000E 07   2.830000E-01 8.630000E 06

ROTOR DATA

STATION	MASS,LBS	POLAR MOM.IN.	TRANSV.MOM.IN.	LENGTH	OUT.DIA(STIFF)	OUT.DIA(MASS)	INNER DIA.
1	6.150000E 00	4.240000E 01	3.100000E 01	1.300000E 00	3.500000E 00	3.500000E 00	3.339000E 00
2	1.610000E 00	0.	0.	1.450000E 00	3.500000E 00	3.600000E 00	3.200000E 00
3	0.	0.	0.	2.370000E 00	3.500000E 00	3.650000E 00	3.062000E 00
4	0.	0.	0.	4.830000E 00	3.500000E 00	3.650000E 00	3.062000E 00
5	0.	0.	0.	1.670000E 00	2.775000E 00	3.250000E 00	2.000000E 00
6	3.100000E 00	1.200000E 01	1.200000E 01	2.100000E 00	3.500000E 00	3.700000E 00	2.000000E 00
7	0.	0.	0.	2.100000E 00	3.500000E 00	3.250000E 00	1.040000E 00
8	3.100000E 00	1.200000E 01	1.200000E 01	1.670000E 00	2.775000E 03	3.250000E 00	2.000000E 00
9	0.	0.	0.	3.490000E 00	3.500000E 00	3.650000E 00	3.062000E 00
10	0.	0.	0.	4.390000E 00	3.500000E 00	3.650000E 00	3.062000E 00
11	9.210000E 00	5.600000E 01	2.500000E 01	0.	1.000000E 00	0.	0.

BEARING STATIONS
4   10

MAGNETIC FORCE DATA

```
    Q(0),FORCE    Q(0),MOMENT
    2.000000E 04  8.820000E 04
```

MATRIX OF COSINE COMPONENTS
```
0.            0.            4.200000E 00   0.
0.            0.            0.            -4.200000E 00
4.200000E 00  0.            0.             0.
0.           -4.200000E 00  0.             0.
```

MATRIX OF SINE COMPONENTS
```
0.            0.            0.            -4.200000E 00
0.            0.           -4.200000E 00   0.
0.           -4.200000E 00  0.             0.
-4.200000E 00  0.           0.             0.
```

BEARING DATA

BEARING AT STATION 4

HARMONIC	KXX	W*BXX	KXY	W*BXY	KYX	W*BYX	KYY	W*BYY
0	1.119158E 05	0.	0.	0.	0.	0.	1.119158E 05	0.
1	1.962623E 05	0.	0.	0.	0.	0.	1.962623E 05	0.
2	1.972986E 05	0.	0.	0.	0.	0.	1.972986E 05	0.
3	1.846317E 05	0.	0.	0.	0.	0.	1.846317E 05	0.
4	1.684247E 05	0.	0.	0.	0.	0.	1.684247E 05	0.
5	1.262724E 05	0.	0.	0.	0.	0.	1.262724E 05	0.
6	2.547524E 05	0.	0.	0.	0.	0.	2.547524E 05	0.
7	2.176094E 05	0.	0.	0.	0.	0.	2.176094E 05	0.
8	1.810601E 05	0.	0.	0.	0.	0.	1.810601E 05	0.
9	1.217425E 05	0.	0.	0.	0.	0.	1.217425E 05	0.
10	-3.497244E 05	0.	0.	0.	0.	0.	-3.497244E 05	0.

BEARING AT STATION 10

HARMONIC	KXX	W*BXX	KXY	W*BXY	KYX	W*BYX	KYY	W*BYY
0	1.119158E 05	0.	0.	0.	0.	0.	1.119158E 05	0.
1	1.962623E 05	0.	0.	0.	0.	0.	1.962623E 05	0.
2	1.972986E 05	0.	0.	0.	0.	0.	1.972986E 05	0.
3	1.846317E 05	0.	0.	0.	0.	0.	1.846317E 05	0.
4	1.684247E 05	0.	0.	0.	0.	0.	1.684247E 05	0.
5	1.262724E 05	0.	0.	0.	0.	0.	1.262724E 05	0.
6	2.547524E 05	0.	0.	0.	0.	0.	2.547524E 05	0.
7	2.176094E 05	0.	0.	0.	0.	0.	2.176094E 05	0.
8	1.810601E 05	0.	0.	0.	0.	0.	1.810601E 05	0.
9	1.217425E 05	0.	0.	0.	0.	0.	1.217425E 05	0.
10	-3.497244E 05	0.	0.	0.	0.	0.	-3.497244E 05	0.

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ROTOR SPEED= 1.030000E 04 RPM

UNDRFLOW AT 53363 IN HQ

CXX	EVEN DETERM.	ODD DETERM.
-0.	2.111015E 00	1.513474E 00
1.000000E 04	2.114852E 00	1.512247E 00
2.000000E 04	2.126397E 00	1.502595E 00
3.000000E 04	2.145743E 00	1.486603E 00
4.000000E 04	2.173045E 00	1.464410E 00
5.000000E 04	2.208537E 00	1.436213E 00
6.000000E 04	2.252498E 00	1.402257E 00
7.000000E 04	2.305294E 00	1.362842E 00
8.000000E 04	2.367357E 00	1.318312E 00
9.000000E 04	2.439197E 00	1.269057E 00
1.000000E 05	2.521408E 00	1.215507E 00
1.100000E 05	2.614667E 00	1.158129E 00
1.200000E 05	2.719745E 00	1.097423E 00
1.300000E 05	2.837510E 00	1.033916E 00
1.400000E 05	2.968945E 00	9.681555E-01
1.500000E 05	3.115132E 00	9.007096E-01
1.600000E 05	3.277292E 00	8.321563E-01
1.700000E 05	3.456775E 00	7.630799E-01
1.800000E 05	3.655079E 00	6.940651E-01
1.900000E 05	3.873851E 00	6.256909E-01
2.000000E 05	4.114905E 00	5.585233E-01
2.100000E 05	4.380256E 00	4.931183E-01
2.200000E 05	4.672125E 00	4.299975E-01
2.300000E 05	4.992890E 00	3.696616E-01
2.400000E 05	5.345258E 00	3.125751E-01
2.500000E 05	5.732106E 00	2.591635E-01
2.600000E 05	6.156636E 00	2.098082E-01
2.700000E 05	6.622316E 00	1.648418E-01
2.800000E 05	7.132986E 00	1.245444E-01
2.900000E 05	7.692774E 00	8.913947E-02
3.000000E 05	8.306188E 00	5.879141E-02
3.100000E 05	8.978231E 00	3.360245E-02
3.200000E 05	9.714280E 00	1.361092E-02
3.300000E 05	1.052014E 01	-1.210047E-03
3.250000E 05	1.010808E 01	5.958361E-03
3.290321E 05	1.043891E 01	5.862245E-07
3.400000E 05	1.140215E 01	-1.095307E-02
3.500000E 05	1.236721E 01	-1.577639E-02
3.600000E 05	1.342293E 01	-1.590296E-02
3.700000E 05	1.457748E 01	-1.161866E-02
3.800000E 05	1.583943E 01	-3.269779E-03
3.900000E 05	1.721817E 01	6.740358E-03
3.850000E 05	1.651351E 01	2.304956E-03
3.830249E 05	1.624360E 01	-3.156500E-06
4.000000E 05	1.872450E 01	2.395571E-02
4.100000E 05	2.037091E 01	4.187267E-02
4.200000E 05	2.216305E 01	6.194603E-02
4.300000E 05	2.412313E 01	8.359595E-02
4.400000E 05	2.626082E 01	1.062157E-01
4.500000E 05	2.859093E 01	1.291802E-01
4.600000E 05	3.112933E 01	1.518559E-01
4.700000E 05	3.389487E 01	1.736104E-01
4.800000E 05	3.690608E 01	1.938243E-01
4.900000E 05	4.018274E 01	2.119022E-01
5.000000E 05	4.374745E 01	2.272860E-01
5.100000E 05	4.762281E 01	2.394674E-01
5.200000E 05	5.183484E 01	2.480018E-01

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```
5.300000E 05    5.640991E 01    2.525327E-01
5.400000E 05    6.137715E 01    2.527559E-01
5.500000E 05    6.676704E 01    2.485348E-01
5.600000E 05    7.261270E 01    2.398153E-01
5.700000E 05    7.894963E 01    2.266910E-01
5.800000E 05    8.581567E 01    2.094089E-01
5.900000E 05    9.325071E 01    1.883841E-01
6.000000E 05    1.012980E 02    1.642147E-01
6.100000E 05    1.100037E 02    1.376969E-01
6.200000E 05    1.194158E 02    1.098385E-01
6.300000E 05    1.295874E 02    8.187246E-02
6.400000E 05    1.405734E 02    5.526939E-02
6.500000E 05    1.524326E 02    3.174912E-02
6.600000E 05    1.652282E 02    1.329130E-02
6.700000E 05    1.790253E 02    2.144265E-03
6.800000E 05    1.938964E 02    8.327640E-04
6.900000E 05    2.099157E 02    1.216391E-02
7.000000E 05    2.271629E 02    3.923128E-02
7.100000E 05    2.457224E 02    8.541692E-02
7.200000E 05    2.656836E 02    1.543913E-01
7.300000E 05    2.871419E 02    2.501109E-01
7.400000E 05    3.101962E 02    3.768126E-01
7.500000E 05    3.349534E 02    5.390065E-01
7.600000E 05    3.615265E 02    7.414649E-01
7.700000E 05    3.900315E 02    9.892078E-01
7.800000E 05    4.205948E 02    1.287487E 00
7.900000E 05    4.533498E 02    1.641763E 00
8.000000E 05    4.884344E 02    2.057687E 00
8.100000E 05    5.259974E 02    2.541064E 00
8.200000E 05    5.661915E 02    3.097830E 00
8.300000E 05    6.091827E 02    3.734009E 00
8.400000E 05    6.551415E 02    4.455679E 00
8.500000E 05    7.042479E 02    5.268929E 00
8.600000E 05    7.566958E 02    6.179804E 00
8.700000E 05    8.126833E 02    7.194261E 00
8.800000E 05    8.724218E 02    8.318111E 00
8.900000E 05    9.361331E 02    9.556956E 00
9.000000E 05    1.004051E 03    1.091613E 01
9.100000E 05    1.076420E 03    1.240063E 01
9.200000E 05    1.153493E 03    1.401503E 01
9.300000E 05    1.235545E 03    1.576344E 01
9.400000E 05    1.322856E 03    1.764940E 01
9.500000E 05    1.415717E 03    1.967578E 01
9.600000E 05    1.514448E 03    2.184475E 01
9.700000E 05    1.619368E 03    2.415767E 01
9.800000E 05    1.730818E 03    2.661496E 01
9.900000E 05    1.849154E 03    2.921608E 01
```
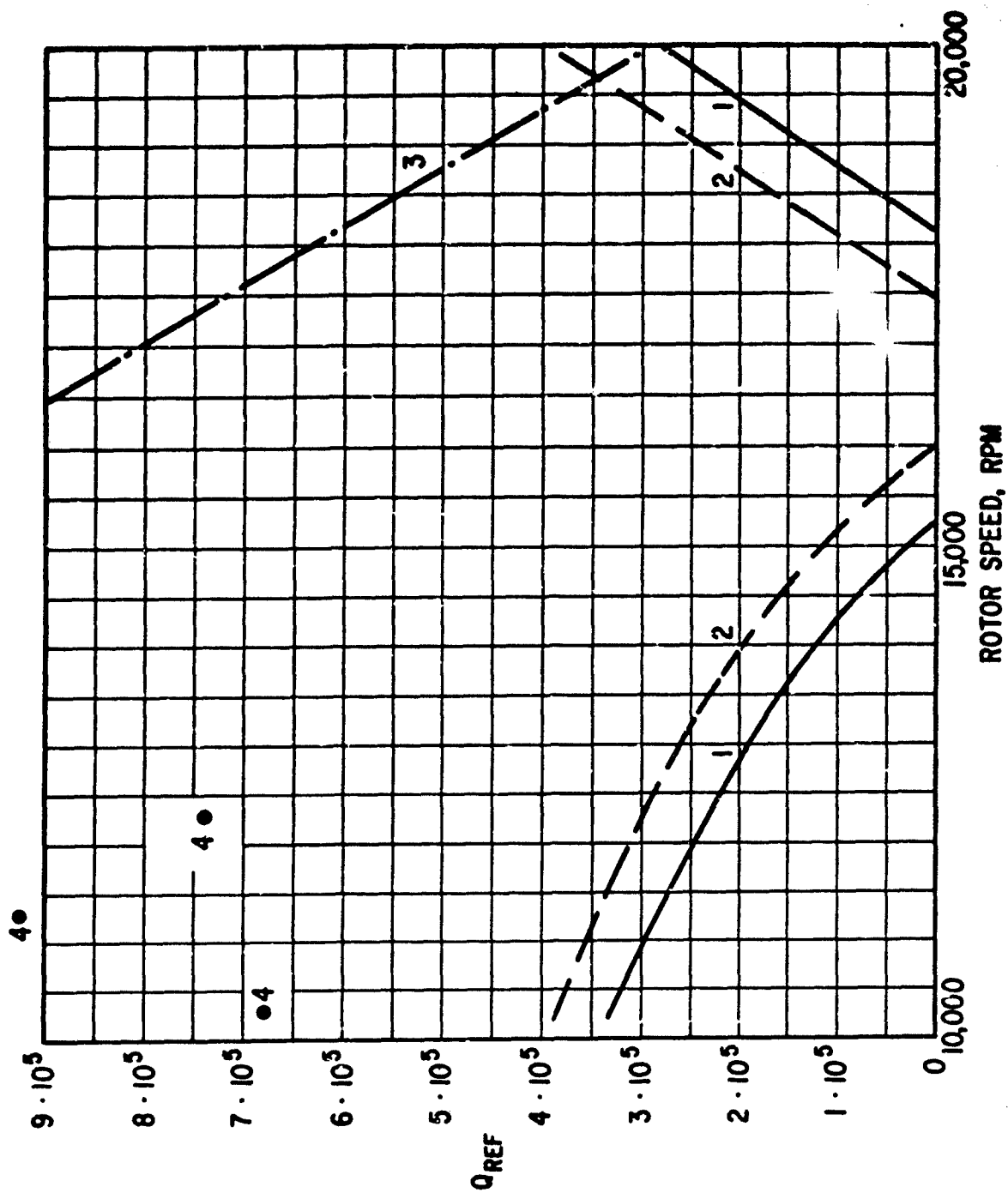
Figure 20   Stability Map for Four Pole Homopolar Generator

APPENDIX XII :Computer Program – The Response of a Rotor with Timevarying
Magnetic Forces

This appendix describes the computer program PN0354: "The Response of a Rotor
with Timevarying Magnetic Forces" and gives the detailed instructions for using
the program. The program is based on the analysis contained in Appendix X
(and Appendix VIII)It calculates the whirl amplitude of an alternator rotor
which is eccentric and misaligned with respect to the axis of the alternator
stator.

The response program has most features in common with the stability program.
Both programs employ the same model of the rotor-bearing system and the form
of the generator magnetic forces is the same for the two programs. Hence,
much of the input data is the same for the two programs and in giving the
instructions for preparation of the input to the response program, reference
will be made to the instructions already given for the stability program in
Appendix XI.

COMPUTER INPUT

An input data form is given in back of this appendix for quick reference when
preparing the computer input data. In the following the more detailed instructions
are given except for those parts which have already been covered in Appendix XI.

Card 1 (72H) Any descriptive text may be given, identifying the calculation.

Card 2 (11I5) This is the "Control" cards whose values control the rest of the
input. It is identical to card 2 of the stability program with a few exceptions,
the major one being that the previous item 3, NQ,is eliminated.Card 2 has 11 values:

1.NS specifies the number of rotor stations (NS $\leq$ 50)
2.NB specifies the number of bearings ($1 \leq NB \leq 10$)
3.KA see Appendix XI
4.KC see Appendix XI

5.NRP   see Appendix XI

6.NPD   see Appendix XI

7.INC   see Appendix XI

8.NH   In the response calculation, the program evaluates the frequency response of the rotor-bearing system at certain discrete frequencies. These frequencies are the harmonics of the magnetic force frequency, i.e. at $0, \Omega, 2\Omega, 3\Omega, 4\Omega$, etc., where Ω is the magnetic force frequency (in the stability calculation the frequencies are $0, \frac{1}{2}\Omega, \frac{2}{2}\Omega, \frac{3}{2}\Omega, \frac{4}{2}\Omega, --$). NH specifies the number of the highest harmonic such that the highest frequency is $(NH)\cdot\Omega$.   NH must be equal to or greater than 1 but it cannot exceed 10.

9.NSP   see Appendix XI, card 2, item 10

10.NDIA   see Appendix XI, card 2, Item 11

11.INP   see Appendix XI, card 2, item 12

Card 3   (1P5E14.6)

See Appendix XI

Rotor Data (8E9.2)

See Appendix XI

Bearing Stations (11I5)

See Appendix XI

Pedestal Data (8E9.2)

See Appendix XI

Whirl Orbit Points in Output (1P5E14.6)

The rotor amplitudes x and y are calculated for each rotor station in the form:

$$x = \sum_{k=0}^{NH} \left[x_{ck} \cos(k\Omega t) - x_{sk} \sin(k\Omega t) \right] \qquad (M.1)$$

$$y = \sum_{k=0}^{NH} \left[y_{ck} \cos(k\Omega t) - y_{sk} \sin(k\Omega t) \right] \qquad (M.2)$$

where Ω is the frequency of the magnetic forces in radians/sec, t is time in seconds and NH is item 8 on card 2. The program output lists x_{ck}, x_{sk}, y_{ck} and y_{sk} for $0 \leq k \leq NH$. However, in order to get the maximum amplitude it is necessary to calculate the whirl orbit whose coordinates, of course, are x and y from eqs. (M.1) and (M.2). This is done by calculating x and y at discrete points along the orbit. Let ω be the angular speed of the rotor. Then:

$$k\Omega t = k\left(\frac{\Omega}{\omega}\right)(\omega t) \qquad \underline{0 \leq k \leq NH} \qquad (M.3)$$

where $\left(\frac{\Omega}{\omega}\right)$ is the fixed ratio between the magnetic force frequency and the speed of the rotor (see the following input card). By varying (ωt) from 0 to 360 degrees, the complete whirl orbit for one shaft revolution can be obtained. The present input card specifies what range of (ωt) is desired and in how big increments the range should be carried. The card has four values:

1. Initial value of (ωt), degrees

2. Final value of (ωt), degrees

3. Increment of (ωt), degrees

4. The lower limit of amplitude values of interest in inches. This item is included because it frequently happens that the amplitudes of the higher harmonics are very small. Then there is no need to include them in the output. This input item specifies what is the smallest amplitude value of interest which may be, say, 10^{-6} inches (1 microinch).

Speed Data (1P5E14.6)

See Appendix XI

Magnetic Force Data

See Appendix XI but disregard all references to Q_{ref}.

Rotor Eccentricity and Misalignment Coordinates (1P4E14.6)

The forces and moments that cause the rotor to whirl, only act when the rotor axis does not coincide with the axis of the alternator stator. Four

coordinates are needed to define the position of the rotor axis with respect to the alternator axis:
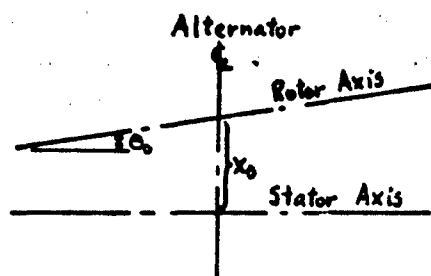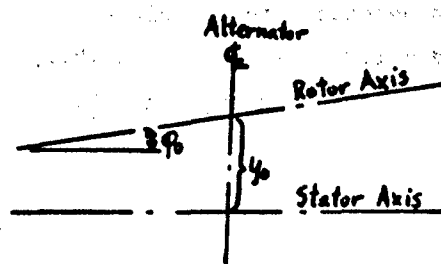
Here, X_0 and Y_0 are the two eccentricity components in the centerplane of the alternator, and Θ_0 and φ_0 are the two corresponding misalignment angles. These values are given on a card which follows next after the magnetic force data. If KC=-1 (card 2, item 4), all four values must be specified:

1. Eccentricity, X_0 of rotor in the centerplane of the alternator, x-direction, inches.

2. Eccentricity, Y_0 of rotor in the centerplane of the alternator, y-direction, inches.

3. Misalignment angle, Θ_0 , of rotor axis with respect to stator axis in the x-plane, radians or inches/inch.

4. Misalignment angle, φ_0 , of rotor axis with respect to stator axis in the y-plane, radians or inches-inch.

If KC $\geq$ 0, only two values can be specified. There will be one card with two values and one of the following two possibilities apply.

KC=0

1. X_0 , inch
2. Y_0 , inch

KC=1

1. Θ_0 , radians
2. φ_0 , radians.

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Bearing Data, Fixed Geometry

This data applies when NPD=0 (item 6, card 2). The instructions for preparing the input are the same as previously given in Appendix XI. However, eq. (L.3) should be changed to read:

$$\frac{\nu}{\omega} = 0, \left(\frac{\Omega}{\omega}\right), 2\left(\frac{\Omega}{\omega}\right), 3\left(\frac{\Omega}{\omega}\right), ----, (NH)\left(\frac{\Omega}{\omega}\right) \qquad (M.4)$$

This does not affect the number or the form of the input cards but only redefines those frequencies at which the bearing coefficients must be evaluated when the lubricant is compressible (INC=1, item 7, card 2).

Bearing Data, Tilting Pad Bearing

This data applies when NPD ≠ 0 (card 2, item 6). The instructions for preparing the input are the same as previously given in Appendix XI. However, when the lubricant is compressible (INC=1, item 7, card 2) those frequencies at which the pad film coefficients are evaluated, are given by eq. (M.4), not eq. (L.3).

COMPUTER OUTPUT

Referring to the later given sample calculation, it is seen that the first three pages of the computer output repeats the input data in the same order in which it is read in by the computer. The only input data which is not repeated in the output, is the card with the speed data and the card specifying the points on the whirl orbit.

Next follow the results of the calculations. First, the rotor speed is identified and, immediately after, the calcualted rotor amplitudes and slopes at the centerplane of the alternator are given. There are 10 columns. The first column identifies the harmonic of the oscillation (i.e. 0, 1, 2,...times the frequency of the magnetic forces), and in the second column are the corresponding frequencies. The 8 remaining columns give the cosine and sine components of the two amplitudes (x and y) and the two slopes ($\Theta = \frac{dx}{dz}$ and $\varphi = \frac{dy}{dz}$). The resulting motion is obtained from eqs. (M1) and (M2) and the analogous ones for Θ and φ . Here, NH is the number of the highest harmonic (in the sample case, NH=5), Ω is the magnetic force frequency, radians/sec, t is time in seconds, and k is the number of the harmonic, given in Column 1. In the output

X (C)= x_{ck}, inch, X(S) = x_{sk}, inch, Y(C) = y_{ck}, inch, Y(S) = y_{sk}, inch, DX/DZ(C) = θ_{ck}, inch/inch, DX/DZ(S) = θ_{sk}, inch/inch, DY/DZ(C) = ϕ_{ck}, inch/inch, and DY/DZ(S) = ϕ_{sk}, inch/inch. These data are given since they result from the most important part of the calculation, namely the solution of the simultaneous equations given by eq. (K.23), Appendix K.

Thereafter there is one page of output for each rotor station. The rotor station is identified first, followed by a list similar to the one described above except that no data are given for θ and ϕ . Next, eqs. (M.1) and (M.2) are used to calculate the rotor motion and the results are given in a five column list under the title "WHIRL ORBIT." For this purpose, eqs. (M.1) and (M.2) are rewritten as:

$$x = \sum_{k=0}^{NH} \left[x_{ck} \cos\left(k\left(\tfrac{\Omega}{\omega}\right)\omega t\right) - x_{sk} \sin\left(k\left(\tfrac{\Omega}{\omega}\right)\omega t\right) \right]$$

$$\tag{M.5}$$

$$y = \sum_{k=0}^{NH} \left[y_{ck} \cos\left(k\left(\tfrac{\Omega}{\omega}\right)\omega t\right) - y_{sk} \sin\left(k\left(\tfrac{\Omega}{\omega}\right)\omega t\right) \right]$$

where ω is the angular speed of the rotor. With $\left(\tfrac{\Omega}{\omega}\right)$ being a fixed ratio, ωt is varied over a range as specified in the input. The value of ωt gives the angle of rotation of the shaft such that as ωt goes from 0 to 360 degrees, the shaft makes one revolution. The first column, titled "SHAFT ROTAT, DEG", specifies the value of ωt . For a given value of ωt , eq. (M.5) can be used directly to calculate the amplitude components x and y, and the values are given in the output in the columns entitled "X" and "Y". They are in inches. Hence, x and y are simply the coordinates of the whirl orbit described by the center of the shaft during one revolution of the shaft. The output also gives the coordinates of the whirl orbit in polar coordinates in the two last columns where:

"AMPLITUDE" = $\sqrt{x^2+y^2}$ inches

"ANGLE X-AMPL" = $\tan^{-1}(y/x)$ degrees

Thus the amplitude gives the "radius" of the orbit, and the angle is the angle from the x-axis to the amplitude direction, positive in the direction of rotor rotation.

SAMPLE CALCULATION

The response of a 4 pole homopolar alternator has been calculated to illustrate the use of the program. The rotor is supported in two gas lubricated tilting pad bearings, and the bearings have four shoes with the static load passing between the pivots of the two bottom shoes. In the centerplane of the alternator (rotor station 7), the center of the rotor coincides with the center of the alternator but the rotor axis is misaligned 0.002 inches/inch with respect to the magnetic axis of the alternator which results in a pulsating force of 84 lbs at a frequency of twice the shaft speed acting on the rotor. With the two first critical speeds at approximately 14,700 rpm and 16,000 rpm it is found that the maximum response occurs at half these speeds. However, only the fundamental harmonic is excited with a significant amplitude and the amplitudes of the higher harmonics are of no practical interest.

PN0354: THE RESPONSE OF A ROTOR WITH TIMEVARYING MAGNETIC FORCES

Card 1 (72H) Text

Card 2 (11I5)

1. NS = Number of rotor stations (NS $\leq$ 50)

2. NB = Number of bearings (NB $\leq$ 10)

3. KA. |KA|= Rotor station number at which magnetic forces act

    $\left.\begin{array}{l} \text{KA} > 0: \text{ forces only, no moments} \\ \text{KA} < 0: \text{ moments only, no forces} \end{array}\right\}$ KC $\geq$ 0

    KA > 0, KC=-1: both forces and moments

4. KC KC=0: the magnetic forces or moments are proportional to amplitudes

    KC=1: the magnetic forces or moments are proportional to slope

    KC=-1: there are both magnetic forces and moments

5. NRP NRP=0: bearing pedestals are rigid, no pedestal input data

    NRP=1: flexible bearing pedestals, pedestal input data required

6. NPD NPD=0: fixed geometry bearings

    NPD $\geq$ 1: number of pads in tilting pad bearing, load between pads

    NPD $\leq$ -1: |NPD|=number of pads in tilting pad bearing, load on pad

7. INC INC=0: bearing lubricant is incompressible

    INC=1: bearing lubricant is compressible

8. NH = Number of frequency harmonics in stability calculation (1 $\leq$ NH $\leq$ 10)

9. NSP = Number of speed ranges with accompanying data (NSP $\geq$ 1)

10. NDIA NDIA=0: rotor impedance matrices not included in output

    NDIA=1: rotor impedance matrices included in output

    NDIA=-1: diagnostic

11. INP INP=0: more input follows, starting from card 1

    INP=1: last set of input data

Card 3 (1P5E14.6)

1. YM = Youngs modulus for shaft material, lbs/in^2

2. DNST = Weight density of shaft material, lbs/in^3

3. SHM = α G, where G is shear modulus, lbs/in^2, and α is shape factor for shear.

Rotor Data (8E9.2)

Give NS cards with 7 numbers on each card:

1. Mass at rotor station, lbs.
2. Polar mass moment of inertia at rotor station, lbs-in^2
3. Transverse mass moment of inertia at rotor station, lbs-in^2
4. Length of shaft section to next station, inch
5. Outer shaft diameter for cross-sectional moment of inertia, inch
6. Outer shaft diameter for shaft mass, inch
7. Inner shaft diameter, inch.

Bearing Stations (11I5)

List the rotor stations at which there are bearings, in total NB stations

Pedestal Data (8E9.2)

This data only applies when NRP=1 (card 2, item 5). Give a total of NB cards with 6 values per card:

1. Pedestal mass, x-direction, lbs.
2. Pedestal stiffness, x-direction, lbs/inch
3. Pedestal damping, x-direction, lbs-sec/inch
4. Pedestal mass, y-direction, lbs.
5. Pedestal stiffness, y-direction, lbs/inch
6. Pedestal damping, y-direction, lbs-sec/inch

Whirl Orbit Points in Output (1P5E14.6)

Give one card with 4 values:

1. Initial value of ωt , degrees
2. Final value of ωt , degrees
3. Increment of ωt , degrees
4. Lower limit for amplitude values of interest, inch

Note: The following data must be repeated NSP-times (Card 2, item 9)

Speed Data (1P5E14.6)

Give one card with 5 values:

1. Initial speed, rpm
2. Final speed, rpm
3. Speed increment, rpm
4. Ratio of magnetic force frequency to rotor speed
5. Scale factor for determinant (set equal to mass of rotor)

Magnetic Force Data

Card (1P5E14.6)

1. Static gradient of magnetic force, Q_o , lbs/in
2. Static gradient of magnetic moment, Q_o' , lbs-inch/radian

Cards (1P4E14.6)

a. If XC=-1 (card 2, item 4), give 8 cards with 4 values per card:

Q_{xx}	Q_{xy}	$Q_{x\theta}$	$Q_{x\varphi}$
Q_{yx}	Q_{yy}	$Q_{y\theta}$	$Q_{y\varphi}$
$Q_{\theta x}$	$Q_{\theta y}$	$Q_{\theta\theta}$	$Q_{\theta\varphi}$
$Q_{\varphi x}$	$Q_{\varphi y}$	$Q_{\varphi\theta}$	$Q_{\varphi\varphi}$
q_{xx}	q_{xy}	$q_{x\theta}$	$q_{x\varphi}$
q_{yx}	q_{yy}	$q_{y\theta}$	$q_{y\varphi}$
$q_{\theta x}$	$q_{\theta y}$	$q_{\theta\theta}$	$q_{\theta\varphi}$
$q_{\varphi x}$	$q_{\varphi y}$	$q_{\varphi\theta}$	$q_{\theta\theta}$

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These are the gradients of the timevarying magnetic forces and moments:

$$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}, q_{xx}, q_{xy}, q_{yx}, q_{yy} \quad \text{in lbs/inch}$$

$$Q_{x\theta}, Q_{x\varphi}, Q_{y\theta}, Q_{y\varphi}, q_{x\theta}, q_{x\varphi}, q_{y\theta}, q_{y\varphi} \quad \text{in lbs/radian}$$

$$Q_{\theta x}, Q_{\theta y}, Q_{\varphi x}, Q_{\varphi y}, q_{\theta x}, q_{\theta y}, q_{\varphi x}, q_{\varphi y} \quad \text{in lbs-inch/inch}$$

$$Q_{\theta\theta}, Q_{\theta\varphi}, Q_{\varphi\theta}, Q_{\varphi\varphi}, q_{\theta\theta}, q_{\theta\varphi}, q_{\varphi\theta}, q_{\varphi\varphi} \quad \text{in lbs-inch/radian}$$

b. If KC=0, give 4 cards with 2 values per card

$KA > 0$		$KA < 0$	
Q_{xx}	Q_{xy}	$Q_{\theta x}$	$Q_{\theta y}$
Q_{yx}	Q_{yy}	$Q_{\varphi x}$	$Q_{\varphi y}$
q_{xx}	q_{xy}	$q_{\theta x}$	$q_{\theta y}$
q_{yx}	q_{yy}	$q_{\varphi x}$	$q_{\varphi y}$

c. If KC=1, give 4 cards with 2 values per card

$KA > 0$		$KA < 0$	
$Q_{x\theta}$	$Q_{x\varphi}$	$Q_{\theta\theta}$	$Q_{\theta\varphi}$
$Q_{y\theta}$	$Q_{y\varphi}$	$Q_{\varphi\theta}$	$Q_{\varphi\varphi}$
$q_{x\theta}$	$q_{x\varphi}$	$q_{\theta\theta}$	$q_{\theta\varphi}$
$q_{y\theta}$	$q_{y\varphi}$	$q_{\varphi\theta}$	$q_{\varphi\varphi}$

<u>Rotor Eccentricity and Misalignment Coordinates (1P4E14.6)</u>

Give one card with either 4 or 2 values on it:

a. If KC=-1:

   1. x_0 , eccentricity of rotor in alternator centerplane, x-direction, inch

   2. y_0 , eccentricity of rotor in alternator centerplane, y-direction, inch

   3. θ_0 , misalignment angle in x-plane, radians or inches/inch

   4. φ_0 , misalignment angle in y-plane, radians or inches/inch

b. If KC=0:

   1. x_0 , inches

   2. y_0 , inches

c. If KC=1:

   1. θ_0 , inches/inch

   2. φ_0 , inches/inch

Bearing Data, Fixed Geometry (8F9.2)

Applies when NPD=0 (card 2, item 6). If the lubricant is incompressible (INC=0; card 2, item 7), give one card per bearing. If the lubricant is compressible (INC=1), give (NH+1)-cards per bearing (NH is item 8, card 2). Each card gives a set of 8 bearing coefficients:

1. Spring coefficient K_{xx}, lbs/inch
2. Damping ωB_{xx}, lbs/inch
3. Spring coefficient K_{xy}, lbs/inch
4. Damping ωB_{xy}, lbs/inch
5. Spring coefficient K_{yx}, lbs/inch
6. Damping ωB_{yx}, lbs/inch
7. Spring coefficient K_{yy}, lbs/inch
8. Damping ωB_{yy}, lbs/inch

Bearing Data, Tilting Pad Bearing

Applies when NPD $\neq$ 0 (card 2, item 6). Define the number NPD1 by:

if NPD $\geq$ 1 (load between pads): $\begin{cases} \text{NPD even, then: } \text{NPD1}=1/2 \cdot \text{NPD} \\ \text{NPD odd, then: } \text{NPD1}=1/2 \cdot (\text{NPD}+1) \end{cases}$

if NPD $\leq$ -1 (load on pad): $\begin{cases} |\text{NPD}| \text{ even, then: } \text{NPD1}=1/2 \cdot |\text{NPD}| +1 \\ |\text{NPD}| \text{ odd, then: } \text{NPD1}=1/2 \cdot (|\text{NPD}| +1) \end{cases}$

NPD1 is the number of pads for which input is required per bearing. If the lubricant is incompressible (INC=0; card 2, item 7), give two cards per pad. If the lubricant is compressible (INC=1), give (NH+2)-cards per pad. In either case the first card is:

(1P5E14.6)

1. Pitch mass moment of inertia divided by the square of the journal radius, lbs
2. Pad mass, lbs
3. Radial stiffness of pivot support, lbs/inch
4. Angle from bearing load line to pivot point, degrees.

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Then follow 1 card if INC=0, or (NH+1)-cards if INC=1, with the 8 dynamic coefficients for the pad:

Card: (8E9.2)

1. Spring coefficient K_{xx}, lbs/inch
2. Damping $\omega\beta_{xx}$, lbs/inch
3. Spring coefficient K_{xy}, lbs/inch
4. Damping $\omega\beta_{xy}$, lbs/inch
5. Spring coefficient K_{yx}, lbs/inch
6. Damping $\omega\beta_{yx}$, lbs/inch
7. Spring coefficient K_{yy}, lbs/inch
8. Damping $\omega\beta_{yy}$, lbs/inch

These (NH+2)-cards must be repeated NPD1 times per bearing, and there must be one complete set for each bearing (there are NB bearings).

```
3-14-1967  J. LUND MECHANICAL TECHNOLOGY INC.
PN354=RESPONSE OF ROTOR WITH MAGNETIC FORCES
COMMON/A/ AKXX(10,11),BCXX(10,11),BKXY(10,11),BCXY(10,11),
1BKYX(10,11),BCYX(10,11),AKYY(10,11),BCYY(10,11),PMIN(10,5),
2PADM(10,5),PADK(10,5),PANG(10,5),DEVN(4,4),WCC(4,1),CMR(4,8),
3CME(4,8),AMR(4,8),AKE(4,8),WR(4,4),WE(4,4),WA(4,4),WB(4,4),
4WC(4,4),WQ(4,4),WSQ(4,4),UR(4,4),UE(4,4),EMR(4,4),EME(4,4),
5XR(4,1),XE(4,1)
COMMON/B/ PKXX(10,5,11),PCXX(10,5,11),PKXY(10,5,11),PCXY(10,5,11),
1PKYX(10,5,11),PCYX(10,5,11),PKYY(10,5,11),PCYY(10,5,11),
2GR(2,8,50),GE(2,8,50),SR(4,4,11),SE(4,4,11),XCS(4,50,11),
3XSS(4,50,11),YCS(4,50,11),YSS(4,50,11)
COMMON/C/ RM(30),RIP(30),RIT(30),RS(30),RW(30),RD(30),RL(30),
1DVXA(30),DVXB(30),DVXC(30),DVXD(30),DVYA(30),DVYB(30),DVYC(30),
2DVYD(30),DVUX(30),DVUY(30),DMUX(30),DMUY(30),DMXA(30),B1(30),
3B2(30),B3(30),B4(30),B5(30),B6(30),B7(30),B8(30),B9(30),B10(30),
4PMX(10),PKX(10),PDX(10),PMY(10),PKY(10),PDY(10),SXX(10),DXX(10),
5SXY(10),DXY(10),SYX(10),DYX(10),SYY(10),DYY(10),LB(10),
6XZ(4),XZ1(4)
COMMON/D/A1,A2,A3,A4,A5,A6,A7,A8,NF,FRQ,KC,SCF1,NDIA,KQ2,NS,KB,
* IK,NH2,EN,AMLM,SPD,SFR1,SFR,WTST,WTIN,WTF, NH1
COMMON/E/KQ1,KQ3,C1,C2,NB,KA,NRP,NPD,INC,NH,NSP,INP,YM,DNST,SHM,
*SPST,SPFN,SPIN,SCF,QZ,QZP,K1,K2, WTFN
COMMON/F/BMXC,BMXS,BMYC,BMYS,VXC,VXS,VYC,VYS,XC,XS,YC,YS,DXC,DXS,
*DYS,C3,C4, NS1, NPD1, NPD2, DYC, NSP1
       WRITE (5,99)                                                     1
190    READ (5,101)                                                     2
       READ (5,102) NS, NB, KA, KC, NRP, NPD, INC, NH, NSP,NDIA,INP     3
       READ (5,101) YM, DNST, SHM                                      15
       WRITE(6,105)                                              27     16
       WRITE(6,103)                                              28     17
       WRITE(6,104)NS,NB,KA,KC,NRP,NPD,INC,NH,NSP,NDIA,INP             18
       WRITE(6,105)                                              30     19
       WRITE(6,102)YM,DNST,SHM                                   31     20
       DNST=DNST/386.069                                        32
       NS1=NS-1                                                 33
       NH1=NH+1                                                 34
       IF(KC)  196,195,195
195    KQ1=4
       KQ2=2
       KQ3=6
       GO TO 197
196    KQ1=8
       KQ2=4
       KQ3=8
197    IF(KA)  198,199,199
198    KB=-KA                                                   36
       GO TO 200                                                37
199    KB=KA                                                    38
200    WRITE(6,110)                                             39     36
       WRITE(6,109)                                             40     37
       DO 203 J=1,NS                                            41
       READ (5,108) RM(J), RIP(J), RIT(J), RL(J), RS(J),RW(J),RD(J)    40
       WRITE(6,107)J,RM(J),RIP(J),RIT(J),RL(J),RS(J),RW(J),RD(J) 43     48
       RM(J)=RM(J)/386.069                                     44
```

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```
      RIP(J)=RIP(J)/386.069                                         45
      RIT(J)=RIT(J)/386.069                                         46
      C1=0.049087383*YM*(RS(J)**4-RD(J)**4)                        47
      RW(J)=0.7853986*DNST*(RW(J)**2-RD(J)**2)                     48
      C2=1.5707960*SHM*(RS(J)**2-RD(J)**2)                         49
      RS(J)=C                                                       50
      IF(C2)  202,202,10.                                           51
  201 RD(J)=C1/C2                                                   52
      GO TO 203                                                     53
  202 RD(J)=0.0                                                     54
  203 CONTINUE                                                      55
      READ (5,101) (LB(J), J=1,NB)                                       84
      WRITE(6,109)                                                  57   91
      WRITE(6,101)(LB(J),J=1,NB)                                    58   92
      IF(NRP)  210,210,204                                          59
  204 WRITE(6,111)                                                  60  100
      WRITE(6,112)                                                  61  101
      DO 205 J=1,NB                                                 62
      READ (5,105) PMX(J), PKX(J), PDX(J), PMY(J), PKY(J), PDY(J)       104
      WRITE(6,107)LB(J),PMX(J),PKX(J),PDX(J),PMY(J),PKY(J),PDY(J)   64  111
      PMX(J)=PMX(J)/386.069                                         65
  205 PMY(J)=PMY(J)/386.069                                         66
  210 IF(NPD)  214,217,211                                          67
  211 IF((NPD/2)*2-NPD) 213,212,212                                 68
  212 NPD1=NPD/2                                                    69
      NPD2=-2                                                       70
      GO TO 217                                                     71
  213 NPD1=(NPD+1)/2                                                72
      NPD2=-1                                                       73
      GO TO 217                                                     74
  214 NPD1=-NPD                                                     75
      IF((NPD1/2)*2-NPD1)  216,215,215                              76
  215 NPD1=NPD1/2+1                                                 77
      NPD2=0                                                        78
      GO TO 217                                                     79
  216 NPD1=(NPD1+1)/2                                               80
      NPD2=1                                                        81
  217 NSP1=1                                                        82
      READ (5,102) WTST, WTFN, WTIN, AMLM                              145
  230 READ (5,102) SPST, SPFN, SPIN, SFR, SCF                          146
      READ (5,102) QZ, QZP                                            147
      WRITE(6,113)                                                     148
      WRITE(6,114)                                                     149
      WRITE(6,102) QZ,QZP                                              150
      WRITE(6,115)                                                     151
      DO 218 I=1,KQ2                                                   
      READ (5,138) (WQ(I,J), J=1,KQ2)                                 154
      WRITE(6,138) (WQ(I,J),J=1,KQ2)                                  159
  218 CONTINUE                                                        
      WRITE(6,126)                                                     166
      DO 219 I=1,KQ2                                                   
      READ (5,138) (WSQ(I,J),J=1,KQ2)                                 169
      WRITE(6,138)(WSQ(I,J),J=1,KQ2)                                  174
  219 CONTINUE                                                        
      READ (5,138) (XZ(I), I=1,KQ2)                                   181
      WRITE(6,129)                                                     188
```

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```
      WRITE(6,13.)(XZ(I),I=.,RQ2)                                    189
      SFR1=0. 047.976*SFN
      WRITE(5,..J)                                               96   196
      IF(INC)  242,241,24L                                       97
 24.  KL=.                                                        98
      GO TO 243                                                  99
 242  KL=NHL                                                    100
 243  DO 255 J=.,NB                                             101
      WRITE(6,.17)LB(J)                                         102   204
      IF(NPD) 25.,244,25.                                       103
 244  WRITE(6,..6)                                              104   207
      DO 245 I=.,KL                                             105
      K2=I-1                                                     106
      READ (5,.05) BKXX(J,I), BCXX(J,I), BKXY(J,I),BCXY(J,I),BKYX(J,I),
     1BCYX(J,I), BKYY(J,I), BCYY(J,I)                                 210
      WRITE(6,.07)K2,BKXX(J,I), BCXX(J,I),BKXY(J,I),BCXY(J,I),BKYX(J,I),
     1BCYX(J,I),BKYY(J,I),BCYY(J,I)                             109
                                                               110   219
 245  CONTINUE                                                  111
      IF(INC)  255,246,255                                     112
 246  DO 247 I=2,NHL                                           113
      BKXX(J,I)=BKXX(J,L)                                       114
      BCXX(J,I)=BCXX(J,L)                                       115
      BKXY(J,I)=BKXY(J,L)                                       116
      BCXY(J,I)=BCXY(J,L)                                       117
      BKYX(J,I)=BKYX(J,L)                                       118
      BCYX(J,I)=BCYX(J,L)                                       119
      BKYY(J,I)=BKYY(J,L)                                       120
 247  BCYY(J,I)=BCYY(J,L)                                       121
      GO TO 255                                                 122
 250  DO 254 K=.,NPDL                                           123
      WRITE(6,1.9)K                                             124   255
      WRITE(6,.2J)                                              125   256
      READ (5,10.) PMIN(J,K),PADM(J,K), PADK(J,K), PANG(J,K)        257
      WRITE(6,.02)PMIN(J,K),PADM(J,K),PADK(J,K),PANG(J,K)       127   262
      PMIN(J,K)=PMIN(J,K)/386.069                               128
      PADM(J,K)=PADM(J,K)/386.069                               129
      PANG(J,K)=0.01745329J*PANG(J,K)                           130
      WRITE(6,1.8)                                              131   273
      DO 25. I=1,KL                                             132
      K2=I-1                                                    133
      READ (5,.06) PKXX(J,K,I),PCXX(J,K,I),PKXY(J,K,I),PCXY(J,K,I),
     1PKYX(J,K,I), PCYX(J,K,I), PKYY(J,K,I),PCYY(J,K,I)             276
      WRITE(6,.07)K2,PKXX(J,K,I),PCXX(J,K,I),PKXY(J,K,I),PCXY(J,K,I),
     1PKYX(J,K,I),PCYX(J,K,I),PKYY(J,K,I),PCYY(J,K,I)          137   285
 25.  CONTINUE                                                  138
      IF(INC) 254,252,254                                       139
 252  DO 253 I=2,NHL                                           140
      PKXX(J,K,I)=PKXX(J,K,L)                                   141
      PCXX(J,K,I)=PCXX(J,K,L)                                   142
      PKXY(J,K,I)=PKXY(J,K,L)                                   143
      PCXY(J,K,I)=PCXY(J,K,L)                                   144
      PKYX(J,K,I)=PKYX(J,K,L)                                   145
      PCYX(J,K,I)=PCYX(J,K,L)                                   146
      PKYY(J,K,I)=PKYY(J,K,L)                                   147
 253  PCYY(J,K,I)=PCYY(J,K,L)                                   148
 254  CONTINUE                                                  149
```

```
255 CONTINUE                                                    150
    SPD=SPST                                                    151
260 WRITE(6,12.)SPD                                            152      323
    SCF1=0.10471976*SPD                                        153
    SCF1=SCF*SCF1*SCF1                                          154
    C1=0.5/SCF1
    DO 261 I=1,KQ2
    DO 261 J=1,KQ2
    SR(I,J,NH1)=0.0
    SE(I,J,NH1)=0.0
    WB(I,J)=C1*WQ(I,J)
261 WC(I,J)=-C1*WSQ(I,J)
    CALL AA2 ($260,$230,$190)                                            339
99  FORMAT (1H1)
100 FORMAT(72H                                                  740
   1                                                            741
101 FORMAT(11I5)                                                742
102 FORMAT(1P5E14.6)                                            743
103 FORMAT(108HOSTATIONS  BEARINGS   MAGN.ST   F+M/F/M   RIG.PED    NO.
   1PADS  COMPRESS  HARMONICS NO.SPEEDS  DIAGNOS    INPUT)
104 FORMAT(I6,10I10)                                            746
105 FORMAT(42H0 YOUNGS MOD.     DENSITY    (SHAPE FACT)*G)      747
106 FORMAT(8E9.2)                                               748
107 FORMAT(I5,1PE16.6,1P7E14.6)                                 749
108 FORMAT(104H STATION      MASS,LBS    POLAR MOM.IN. TRANSV.MOM.IN     L  750
   1ENGTH   OUT.DIA(STIFF) OUT.DIA(MASS)   INNER DIA.)                     751
109 FORMAT(17HOBEARING STATIONS)                                752
110 FORMAT(11HOROTOR DATA)                                      753
111 FORMAT(14HOPEDESTAL DATA)                                   754
112 FORMAT(89H STATION    MASS-X,LBS    STIFFNESS-X    DAMPING-X    MASS  755
   1-Y,LBS    STIFFNESS-Y    DAMPING-Y)                                    756
113 FORMAT(20H1MAGNETIC FORCE DATA)                             757
114 FORMAT(27H0  Q(0),FORCE    Q(0),MOMENT)
115 FORMAT(28H0MATRIX OF COSINE COMPONENTS)
116 FORMAT(//13HOBEARING DATA)                                  760
117 FORMAT(19H0BEARING AT STATION,I3)                           761
118 FORMAT(9H0HARMONIC4X3HKXX10X5HW*BXX10X3HKXY10X5HW*BXY10X3HKYX10X5H  762
   1W*BYX10X3HKYY10X5HW*BYY)                                              763
119 FORMAT(8H0PAD NO.,I3)                                       764
120 FORMAT(55H PITCH MOM.IN.    PAD MASS   PIVOT STIFFN.  PIVOT ANGLE)  765
121 FORMAT(13H ROTOR SPEED=,1PE13.6,4H RPM)                     766
122 FORMAT(//13H0HARMONIC NO.,I3,11H,FREQUENCY=,1PE13.6,8H RAD/SEC)     767
123 FORMAT(21H0BEARING COEFFICIENTS)                            768
124 FORMAT(8H STATION5X3HKXX9X7HFRQ*BXX9X3HKXY9X7HFRQ*BXY9X3HKYX9X7HFR  769
   1Q*BYX9X3HKYY9X7HFRQ*BYY)                                             770
125 FORMAT(31H ROTOR-BEARING IMPEDANCE MATRIX)                  771
126 FORMAT(26H0MATRIX OF SINE COMPONENTS)
127 FORMAT(10H0REAL PART)
128 FORMAT(15H0IMAGINARY PART)
129 FORMAT(44H0COORDINATES OF STATIC ROTOR ECCENTRICITY)
130 FORMAT(34H0REAL PART OF A-MATRIX IS SINGULAR)
131 FORMAT(10H HARMONIC=,I3,12H  FREQUENCY=,1PE13.6)
132 FORMAT(18H0ROTOR STATION NO.,I2)
133 FORMAT(25H0FINAL MATRIX IS SINGULAR)
134 FORMAT(49H0AMPLITUDES AT ROTOR STATION WITH MAGNETIC FORCES)
135 FORMAT(23H HARMONIC  FREQUENCY7X4HX(C)10X4HX(S)10X4HY(C)10X4HY(S))
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136 FORMAT(///.2HOWHIRL ORBIT)
137 FORMAT( 6H SHAFT ROTAT,DEG5X1HX13X1HY9X9HAMPLITUDE4X12HANGLE X-AMP
   1L)
138 FORMAT(1P4E14.6)
139 FORMAT(2 H HARMONIC   FREQUENCY5X4HX(C)8X4HX(S)8X4HY(C)8X4HY(S)6X8
   1HDX/DZ(C)4X8HDX/DZ(S)4X8HDY/DZ(C)4X8HDY/DZ(S))
140 FORMAT(42HOREAL PART OF IMPEDANCE MATRIX IS SINGULAR)
141 FORMAT(I5, PE16.4,1P8E12.4)
142 FORMAT(37HOREAL PART OF S-MATRIX IS SINGULAR,K=,I3)
143 FORMAT(25HJINVERSE IMPEDANCE MATRIX)
144 FORMAT(54HJINFLUENCE COEFFICIENT MATRIX E (X(J)=E*F) FOR STATION,I
   13)
145 FORMAT(46HJINFLUENCE MATRIX A (X(J)=A*X(KB)) FOR STATION,I3)
146 FORMAT(35HJINVERSION MATRIX FOR A IS SINGULAR)
147 FORMAT(35HJINVERSION MATRIX FOR E IS SINGULAR)
148 FORMAT(38HJINVERSION MATRIX FOR S IS SINGULAR,K=I3)
    STOP
    END
```

```
      SUBROUTINE AA2 (*,*,*)
      COMMON/A/ BKXX(10,11),BCXX(10,11),BKXY(10,11),BCXY(10,11),
     1BKYX(10,11),BCYX(10,11),BKYY(10,11),BCYY(10,11),PMIN(10,5),
     2PAOM(10,5),PAUK(10,5),PANG(10,5),DEVN(4,4),WCC(4,1),CMR(4,8),
     3CME(4,8),AMR(4,8),AME(4,8),WR(4,4),WE(4,4),WA(4,4),WB(4,4),
     4WC(4,4),WQ(4,4),WSQ(4,4),UR(4,4),UE(4,4),EMR(4,4),EME(4,4),
     5XR(4,11),XE(4,11)
      COMMON/B/ PKXX(10,5,11),PCXX(10,5,11),PKXY(10,5,11),PCXY(10,5,11),
     1PKYX(10,5,11),PCYX(10,5,11),PKYY(10,5,11),PCYY(10,5,11),
     2GR(2,8,50),GE(2,8,50),SR(4,4,11),SE(4,4,11),XCS(4,50,11),
     3XSS(4,50,11),YCS(4,50,11),YSS(4,50,11)
      COMMON/C/ RM(30),RIP(30),RIT(30),RS(30),RW(30),RD(30),RL(30),
     1DVXA(30),DVXB(30),DVXC(30),DVXD(30),DVYA(30),DVYB(30),DVYC(30),
     2DVYD(30),DVUX(30),DVUY(30),DMUX(30),DMUY(30),DMXA(30),B1(30),
     3B2(30),B3(30),B4(30),B5(30),B6(30),B7(30),B8(30),B9(30),B10(30),
     4PMX(10),PKX(10),PDX(10),PMY(10),PKY(10),PDY(10),SXX(10),DXX(10),
     5SXY(10),DXY(10),SYX(10),DYX(10),SYY(10),DYY(10),LB(10),
     6XZ(4),XZ1(4)
      COMMON/D/A1,A2,A3,A4,A5,A6,A7,A8,NF,FRQ,KC,SCF1,NDIA,KQ2,NS,KB,
     * IK,NH2,HN,AHLM,SPD,SFR1,SFR,WTST,WTIN,WTF, NH1
      COMMON/E/KQ1,KQ3,C1,C2,NB,KA,NRP,NPD,INC,NH,NSP,INP,YM,DNST,SHM,
     *SPST,SPFN,SPIN,SCF,QZ,QZP,K1,K2, WTFN
      COMMON/F/BMXC,BMXS,BMYC,BMYS,VXC,VXS,VYC,VYS,XC,XS,YC,YS,DXC,DXS,
     *DYS,C3,C4, NS1, NPD1, NPD2, DYC, NSP1
      DO 530 IK=1,NH1
      IH=NH1+1-IK
      NF=IH-1
      HN=NF                                                          157
      FRQ=HN*SPD*SFR1                                                158
      FQ2=FRQ*FRQ                                                    159
      HN1=HN*SFR
      IF(NCIA) 401,402,401                                          161
  401 WRITE(6,122)NF,FRQ                                            162      8
      WRITE(6,123)                                                  163      9
      WRITE(6,124)                                                  164     10
  402 DO 425 J=1,NB                                                 165
      IF(NPD)404,403,404                                            166
  403 D1=BKXX(J,IH)                                                 167
      D2=BCXX(J,IH)*HN1                                             168
      D3=BKXY(J,IH)                                                 169
      D4=BCXY(J,IH)*HN1                                             170
      D5=BKYX(J,IH)                                                 171
      D6=BCYX(J,IH)*HN1                                             172
      D7=BKYY(J,IH)                                                 173
      D8=BCYY(J,IH)*HN1                                             174
      GO TO 416                                                     175
  404 D1=0.0                                                        176
      D2=0.0                                                        177
      D3=0.0                                                        178
      D4=0.0                                                        179
      D5=0.0                                                        180
      D6=0.0                                                        181
      D7=0.0                                                        182
      D8=0.0                                                        183
      DO 415 I=1,NPD1                                               184
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C1=FO2*PMIN(J,I)                                    185
A1=PKXX(J,I,IH)                                     186
A2=PCXX(J,I,IH)*HN.                                 187
A3=PKXY(J,I,IH)                                     188
A4=PCXY(J,I,IH)*HN.                                 189
A5=PKYX(J,I,IH)                                     190
A6=PCYX(J,I,IH)*HN.                                 191
A7=PKYY(J,I,IH)                                     192
A8=PCYY(J,I,IH)*HN.                                 193
C2=AT-C.                                            194
C3=PADK(J,I)-FO2*PADM(J,I)                          195
C4=A.+C1                                            196
C5=C4*C2-A.*A8-A3*A5+A4*A6                          197
C6=C4*A8+C2*A2-A3*A6-A5*A4                          198
C7=C5*C5+C2*C6                                      199
C11R=C3*(C2*C2+C6*A8)/C7                            200
C11E=C3*(C.*A8-C5*C2)/C7                            201
C12R=C1*(C5*A3+C6*A4)/C7                            202
C12E=C1*(C5*A4-C6*A3)/C7                            203
C21R=-C3*(C5*A5+C6*A6)/C7                           204
C21E=C3*(C6*A5-C5*A6)/C7                            205
C22R=-C.*(C5*C4+C6*A2)/C7                           206
C22E=C1*(C6*C4-C5*A2)/C7                            207
DKXX=C11R*A1-C11E*A2+C21R*A3-C21E*A4                208
DCXX=C11R*A2+C11E*A1+C21R*A4+C21E*A3                209
DKXY=C12R*A1-C12E*A2+C22R*A3-C22E*A4                210
DCXY=C12R*A1+C12E*A1+C22R*A4+C22E*A3                211
DKYX=C11R*A5-C11E*A6+C21R*A7-C21E*A8                212
DCYX=C11R*A6+C11E*A5+C21R*A8+C21E*A7                213
DKYY=C12R*A5-C12E*A6+C22R*A7-C22E*A8                214
DCYY=C12R*A6+C12E*A5+C22R*A8+C22E*A7                215
C3=PANG(J,I)                                        216
C1=COS(C3)                                          217    40
C2=SIN(C3)                                          218    41
C4=C1*C.                                            219
C5=C2*C.                                            220
A1=DKXX*C4+DKYY*C5                                  221
A2=DCXX*C4+DCYY*C5                                  222
A3=DKXY*C4-DKYX*C5                                  223
A4=DCXY*C4-DCYX*C5                                  224
A5=DKYX*C4-DKXY*C5                                  225
A6=DCYX*C4-DCXY*C5                                  226
A7=DKYY*C4+DKXX*C5                                  227
A8=DCYY*C4+DCXX*C5                                  228
      IF(NPD2+1)  4.0,405,405                       229
405 IF(I-1)  406,406,407                            230
406 IF(NPD2)  409,409,4.0                           231
407 IF(I-NPD1) 4.0,408,408                          232
408 IF(NPD2)  4.0,409,409                           233
409 D1=D1+A.                                        234
    D2=D2+A2                                        235
    D3=D3+A3                                        236
    D4=D4+A4                                        237
    D5=D5+A5                                        238
    D6=D6+A6                                        239
    D7=D7+A7                                        240
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      D8=D8+A8                                                          241
      GO TO 415                                                         242
410   D1=D1+A1+A1                                                       243
      D2=D2+A2+A2                                                       244
      D3=D3+A3+A3                                                       245
      D4=D4+A4+A4                                                       246
      D5=D5+A5+A5                                                       247
      D6=D6+A6+A6                                                       248
      D7=D7+A7+A7                                                       249
      D8=D8+A8+A8                                                       250
415   CONTINUE                                                          251
416   IF(NDIA)   417,420,417                                            252
417   WRITE(6,107)LB(J),D1,D2,D3,D4,D5,D6,D7,D8                         253      58
420   IF(NRP) 421,421,422                                               254
421   SXX(J)=D1                                                         255
      DXX(J)=D2                                                         256
      SXY(J)=D3                                                         257
      DXY(J)=D4                                                         258
      SYX(J)=D5                                                         259
      DYX(J)=D6                                                         260
      SYY(J)=D7                                                         261
      DYY(J)=D8                                                         262
      GO TO 425                                                         263
422   C1=PKX(J)-FQ2*PMX(J)                                             264
      C2=PKY(J)-FQ2*PMY(J)                                             265
      C3=FRQ*PDX(J)                                                     266
      C4=FRQ*PDY(J)                                                     267
      C5=D1+C1                                                          268
      C6=D7+C2                                                          269
      C7=D2+C3                                                          270
      C8=D8+C4                                                          271
      A1=C5*C6-C7*C8-D3*D5+D4*D6                                        272
      A2=C5*C8+C6*C7-D3*D6-D4*D5                                        273
      C9=A1*A1+A2*A2                                                    274
      A3=C1*C6-C3*C8                                                    275
      A4=C1*C8+C3*C6                                                    276
      C11R=(A3*A1+A4*A2)/C9                                             277
      C11E=(A4*A1-A3*A2)/C9                                             278
      A3=C2*D3-C4*D4                                                    279
      A4=C2*D4+C4*D3                                                    280
      C12R=-(A3*A1+A4*A2)/C9                                            281
      C12E=-(A4*A1-A3*A2)/C9                                            282
      A3=C1*D5-C3*D6                                                    283
      A4=C1*D6+C3*D5                                                    284
      C21R=-(A3*A1+A4*A2)/C9                                            285
      C21E=-(A4*A1-A3*A2)/C9                                            286
      A3=C2*C5-C4*C7                                                    287
      A4=C2*C7+C4*C5                                                    288
      C22R=(A3*A1+A4*A2)/C9                                             289
      C22E=(A4*A1-A3*A2)/C9                                             290
      SXX(J)=C11R*D1-C11E*D2+C21R*D3-C21E*D4                            291
      DXX(J)=C11R*D2+C11E*D1+C21R*D4+C21E*D3                            292
      SXY(J)=C12R*D1-C12E*D2+C22R*D3-C22E*D4                            293
      DXY(J)=C12R*D2+C12E*D1+C22R*D4+C22E*D3                            294
      SYX(J)=C11R*D5-C11E*D6+C21R*D7-C21E*D8                            295
      DYX(J)=C11R*D6+C11E*D5+C21R*D8+C21E*D7                            296
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      SYY(J)=C.29*D5-C.23*D6+C?2H*C7-C22E*D8                      297
      DYY(J)=C.2H*D6+C.29*D5+C22R*D8+C22E*D7                      298
  425 CONTINUE                                                    299
      DO 449 J=1,NS.                                              300
      C1=RS(J)                                                    301
      C2=FC2*RW(J)                                                302
      C3=RO(J)                                                    303
      C4=C2/C.                                                    304
      C5=SORT(C4)                                                 305
      C5=SQRT(C5)                                              303    94
      C7=RL(J)                                                 306    95
      IF(C6*C7-J.03)  44.,441,442                                307
  44. C8=C2*C7                                                    308
      B1(J)=1.C                                                   309
      B2(J)=1.C                                                   310
      B3(J)=C7                                                    311
      B6(J)=C7/C.                                                 312
      B4(J)=B6(J)/2.0*C7                                          313
      B7(J)=B4(J)/3.0*C7-C3*C7/C1*2.0                             314
      B5(J)=C2*B7(J)                                              315
      B8(J)=C8                                                    316
      B9(J)=C8/2.0*C7                                             317
      B10(J)=B9(J)/3.0*C7                                         318
      GO TO 449                                                   319
  442 C8=C3*C3*C4                                                 320
      C9=C3*C4                                                    321
      IF(C8-0.00.2)   443,443,444                                322
  443 C8=1.0+0.5*C8                                               323
      GO TO 445                                                   324
  444 C8=SQRT(1.0+C8)                                          326   119
  445 A5=C5*(C8-C9)                                               327
      A6=C5*(C8+C9)                                               328
      A9=A5+A6                                                    329
      A3=SORT(A5)                                              330   121
      A4=SQRT(A6)                                              331   122
      A7=A3*A5                                                    332
      A8=A4*A6                                                    333
      A1=A3*C7                                                    334
      A2=A4*C7                                                    335
      TWOPI = 2.*3.1415926
      D1= COS(AMOD(A2,TWOPI))/A9                                     123
      D2= SIN(AMOD(A2,TWOPI))/A9                                     124
      D5=EXP(A1)                                               338   125
      D4=1.0/D5                                                   339
      D3=0.5*(D5+D4)/A9                                           340
      D4=0.5*(D5-D4)/A9                                           341
      B1(J)=A5*D2+A6*D1                                           342
      B2(J)=A6*D3+A5*D1                                           343
      B3(J)=A3*D4+A4*D2                                           344
      B8(J)=C2*B3(J)                                              345
      B4(J)=(D3-D1)/C1                                            346
      B9(J)=C2*(D3-D1)                                            347
      B5(J)=C5*(A4*D4-A3*D2)                                      348
      B10(J)=C1*B5(J)                                             349
      C8=C1*C5                                                    350
      B6(J)=(A8*D4+A7*D2)/C8                                      351
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        B7(J)=(A7*D4-A8*D2)/C2
    449 CONTINUE                                                   352
        DO 455 J=1,NS                                             353
        C1=FQ2*RM(J)                                              354
        DVXA(J)=C1                                                355
        DVYA(J)=C1                                                356
        DVXB(J)=0.J                                               357
        DVXC(J)=0.0                                               358
        DVXD(J)=0.0                                               359
        DVYB(J)=0.J                                               360
        DVYC(J)=0.0                                               361
        DVYD(J)=0.J                                               362
        DMXA(J)=FQ2*RIT(J)                                        363
        DVUX(J)=0.J                                               364
        DVUY(J)=0.J                                               365
        DMUX(J)=0.0                                               366
    455 DMUY(J)=0.0                                               367
        DO 456 J=1,NB                                             368
        K1=LE(J)                                                  369
        DVXA(K1)=DVXA(K1)-SXX(J)                                  370
        DVXB(K1)=DXX(J)                                           371
        DVXC(K1)=SXY(J)                                           372
        DVXD(K1)=DXY(J)                                           373
        DVYA(K1)=DVYA(K1)-SYY(J)                                  374
        DVYB(K1)=DYY(J)                                           375
        DVYC(K1)=SYX(J)                                           376
    456 DVYD(K1)=DYX(J)                                           377
        DVXA(KB)=DVXA(KB)+QZ                                      378
        DVYA(KB)=DVYA(KB)+QZ                                      379
        DMXA(KB)=DMXA(KB)+QZP                                     380
        CALL BB2                                                  381
        DO 486 J=1,4                                                      189
        DO 486 I=1,4                                              472
        WR(I,J)=AMR(I,J)                                          473
    486 WE(I,J)=AME(I,J)                                          474
        CALL MATINV(WR,4,WCC,0,DVN,ID)                            475
        GO TO(481,460),ID                                                202
    460 WRITE(6,130)
        WRITE(6,133)NF,FRQ                                               204
        GO TO 530                                                       205
    481 IF(NF) 457,457,482
    457 DO 458 I=1,4
        DO 458 J=1,4
        AME(I,J)=0.0
        UR(I,J)=WR(I,J)
    458 UE(I,J)=0.0
        GO TO 459
    482 CALL MATINV(WE,4,WCC,0,DVN,ID)
        GO TO(483,457),ID                                                223
    483 DO 488 I=1,4
        DO 488 J=1,4
        C1=0.0                                                   479
        DO 487 K=1,4                                              480
    487 C1=C1+WR(I,K)*AME(K,J)+WE(I,K)*AMR(K,J)                   481
    488 WA(I,J)=C1                                                482
        CALL MATINV(WA,4,WCC,0,DVN,ID)                            483
                                                                         242
```

```
      GO TO (703,702),IC
  702 WRITE(6,24)                                                        244
      WRITE(6,13)INF,FRQ                                                 245
      GO TO 303
  703 DO 490 I=1,4
      DO 490 J=1,4                                                       485
      C1=0.0                                                             486
      C2=0.0                                                             487
      DO 489 K=1,4                                                       488
      C1=C1+WA(I,K)*WE(K,J)                                              489
  489 C2=C2-WA(I,K)*WR(K,J)                                              490
      UR(I,J)=C1                                                         491
  490 UE(I,J)=C2                                                         492
  459 DO 492 I=1,4
      DO 492  J=1,KQ3
      C1=0.0
      C2=0.0
      DO 491 K=1,4
      C1=C1-UR(I,K)*AMR(K,J)+UE(I,K)*AME(K,J)
  491 C2=C1-UR(I,K)*AME(K,J)-UE(I,K)*AMR(K,J)
      WR(I,J-4)=C1
  492 WE(I,J-4)=C2
      DO 494 I=1,4
      DO 494  J=1,KQ2
      C1=CMR(I,J+4)
      C2=CME(I,J+4)
      DO 493 K=1,4
      C1=C1+CMR(I,K)*WR(K,J)-CME(I,K)*WE(K,J)
  493 C2=C2+CMR(I,K)*WE(K,J)+CME(I,K)*WR(K,J)
      UR(I,J)=C1
  494 UE(I,J)=C2
      IF(NDIA)  704,705,705
  704 WRITE(6,143)                                                       311
      WRITE(6,127)                                                       312
      WRITE(6,138)((UR(I,J),J=1,KQ2),I=1,4)                             313
      WRITE(6,125)                                                       321
      WRITE(6,138)((UE(I,J),J=1,KQ2),I=1,4)                             322
  705 IF(KC)  613,610,613
  610 DO 497 I=1,2
      DO 497 J=1,2                                                       512
      IF(KC)  613,495,496
  495 EMR(I,J)=UR(I,J)                                                   514
      EME(I,J)=UE(I,J)                                                   515
      GO TO 497                                                          516
  496 EMR(I,J)=UR(I+2,J)                                                 517
      EME(I,J)=UE(I+2,J)                                                 518
  497 CONTINUE                                                           519
      C1=EMR(1,1)*EMR(2,2)-EMR(1,2)*EMR(2,1)-EME(1,1)*EME(2,2)+          520
     1EME(1,2)*EME(2,1)                                                  521
      C2=EMR(1,1)*EME(2,2)+EMR(2,2)*EME(1,1)-EMR(1,2)*EME(2,1)-          522
     1EMR(2,1)*EME(1,2)                                                  523
      C3=(C1*C1+C2*C2)*SCF1                                              524
      C1=C1/C3                                                           525
      C2=C2/C3                                                           526
      UR(1,1)    =C1*EMR(2,2)+C2*EME(2,2)
      UR(1,2)    =-C1*EMR(1,2)-C2*EME(1,2)
```

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```
       UR(2,1)    =-C1*EMR(2,1)-C2*EME(2,1)
       UR(2,2)    = C1*EMR(1,1)+C2*EME(1,1)
       UE(1,1)    = C1*EME(2,2)-C2*EMR(2,2)
       UE(1,2)    =-C1*EME(1,2)+C2*EMR(1,2)
       UE(2,1)    =-C1*EME(2,1)+C2*EMR(2,1)
       UE(2,2)    = C1*EME(1,1)-C2*EMR(1,1)
       GO TO 623
611 DO 612 I=1,4
    DO 612 J=1,4
    EMR(I,J)=UR(I,J)
612 EME(I,J)=UE(I,J)
    CALL MATINV(EMR,4,WCC,C,DVN,ID)                                  365
    GO TO(614,613),ID
613 WRITE(6,14J)                                                     367
    WRITE(6,13.) NF,FRQ                                              368
    GO TC 530
614 IF(NF)  615,615,617
615 DO 616 I=1,4
    DO 616 J=1,4
    UR(I,J)=EMR(I,J)/SCF1
616 UE(I,J)=0.0
    GO TO 623
617 CALL MATINV(EME,4,WCC,C,DVN,ID)                                  385
    GO TO(618,615),ID
618 DO 620 I=1,4
    DO 620 J=1,4
    C1=0.0
    DO 619 K=1,4
619 C1=C1+EMR(I,K)*UE(K,J)+EME(I,K)*UR(K,J)
620 WA(I,J)=C1
    CALL MATINV(WA,4,WCC,0,DVN,ID)                                   404
    GO TO (707,706),ID
706 WRITE(6,147)                                                     406
    WRITE(6,13.)NF,FRQ                                               407
    GO TO 530
707 DO 622 I=1,4
    DO 622 J=1,4
    C1=0.0
    C2=0.0
    DO 621 K=1,4
    C1=C1+WA(I,K)*EME(K,J)
621 C2=C2-WA(I,K)*EMR(K,J)
    UR(I,J)=C1/SCF1
622 UE(I,J)=C2/SCF1
623 IF(NDIA)  498,499,498
498 WRITE(6,125)                                                     429
    WRITE(6,127)                                                     430
    WRITE(6,136)((UR(I,J),J=1,4),I=1,4)                              431
    WRITE(6,126)                                                     439
    WRITE(6,136)((UE(I,J),J=1,4),I=1,4)                              440
499 DO 504 J=1,NS
    DO 501 I=1,2
    DO 501 K=1,KQ2
    C1=GR(I,K+4,J)
    C2=GE(I,K+4,J)
    DO 500 L=1,4
```

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```
      C. =C +GR(I,L,J)+WR(L,K)-GE(I,L,J)+WE(L,K)
303   C2=C.+GR(I,L,J)+WE(L,K)+GE(I,L,J)+WR(L,K)
      CMR(I,K)=C
30.   CME(I,K)=C.
      DO 303 I=.,KQ2
      C. =J.J
      C2=J.J
      C3=C..
      C4=J.0
      DO 302 K=.,KQ2
      C.=C +CMR( ,K)+UR(K,I)-CME(.,K)+UL(K,I)
      C2=C.+CMR( ,K)+UL(K,I)+CME(.,K)+UR(K,I)
      C3=C +CMR(.,K)+UR(K,I)-CME(.,K)+UL(K,I)
302   C4=C4+CMR(.,K)+UL(K,I)+CME(.,K)+UR(K,I)
      XCS(I,J,IH)=C.+SCF
      XSS(I,J,IH)=C2+SCF
      YCS(I,J,IH)=C3+SCF
303   YSS(I,J,IH)=C4+SCF
      IF(NDI4)  708,504,..+
708   IF(J-KB)  504,70.+..
70.   WRIT-(..,44)J                                                        505
      WRITE(6,127)                                                         506
      WRITE(6,.35)((CMR(I,K),K=.,KQ.),I=.,2)                               507
      WRITE(5,.2.)                                                         515
      WRITE(6,.35)((CME(I,K),K=.,KQ.),I=1,2)                               516
      WRITE(5,.45)J                                                        524
      WRITE(6,.27)                                                         525
      WRITE(6,.35)(XCS(I,J,IH),I=.,KQ2)                                    526
      WRITE(6,.36)(YCS(I,J,IH),I=.,KQ2)                                    531
      WRITE(6,.2.)                                                         536
      WRITE(6,.35)(XSS(I,J,IH),I=.,KQ2)                                    537
      WRITE(6,.36)(YSS(I,J,IH),I=.,KQ2)                                    542
504   CONTINUE
      IF(NF)  525,525,3.5
505   DO 507 I=1,KQ2
      DO 507 J=.,KQ2
      C.=UR(I,J)
      C2=UE(I,J)
      DO 506 K=.,KQ.
      C.=C.+WB(I,K)+SR(K,J,IH)-WC(I,K)+SE(K,J,IH)
506   C2=C2+WB(I,K)+SE(K,J,IH)+WC(I,K)+SR(K,J,IH)
      WR(I,J)=C.
507   WE(I,J)=C2
      IF(KC)  51.,508,508
508   C1=WR(.,1)+WR(2,2)-WR(1,2)+WR(2,1)-WE(.,.)+WE(2,2)+WE(1,2)+WE(2,1)   601
      C2=WR(.,1)+WE(2,2)+WR(2,2)+WE(.,.)-WR(.,2)+WE(2,1)-WR(2,1)+WE(1,2)   602
      C3=C.+C.+C.+C2                                                       603
      C.=-C./C3                                                            604
      C2=-C2/C3                                                            605
      EMR(.,.)=C.+WR(2,2)+C.+WE(.,2)                                       606
      EME(1,.)=C.+WE(2,2)-C2+WR(2,2)                                       607
      EMR(1,2)=-C.+WR(1,2)+C.+WE(.,2)                                      608
      EME(1,2)=C2+WR(1,2)-C.+WE(.,2)                                       609
      EMR(2,.)=-C.+WR(2,.)-C2+WE(.,.)                                      61.
      EME(2,.)=C2+WR(2,.)-C.+WE(2,.)                                       61.
      EMR(2,2)=C +WR(.,.)+C2+WE(.,.)                                       6.2
```

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```
      EME(2,2)=C *WE(1,1)-C2*WR(1,1)                            613
      GO TO 522
511   DO 512 I=1,4
      DO 512 J=1,4
      UR(I,J)=WR(I,J)
512   UE(I,J)=WE(I,J)
      CALL MATINV(UR,4,WCC,0,DVN,ID)                            589
      GO TO (514,513),ID
513   WRITE(6,14_)NF                                            591
      GO TO 530
514   CALL MATINV(UE,4,WCC,0,DVN,ID)                            594
      GO TO (517,515),ID
515   DO 516 I=1,4
      DO 516 J=1,4
      EMR(I,J)=-UR(I,J)
516   EME(I,J)=0.0
      GO TO 522
517   DO 519 I=1,4
      DO 519 J=1,4
      C1=0.0
      DO 518 K=1,4
518   C1=C1+UR(I,K)*WE(K,J)+UE(I,K)*WR(K,J)
519   WA(I,J)=C1
      CALL MATINV(WA,4,WCC,0,DVN,ID)                            625
      GO TO (710,709),ID
709   WRITE(6,145)NF                                            627
      GO TO 530
710   DO 521 I=1,4
      DO 521 J=1,4
      C1=0.0
      C2=0.0
      DO 520 K=1,4
      C1=C1+WA(I,K)*UE(K,J)
520   C2=C2-WA(I,K)*UR(K,J)
      EMR(I,J)=-C1
521   EME(I,J)=-C2
522   DO 524 I=1,KQ2
      DO 524 J=1,KQ2
      C1=0.0                                                    616
      C2=0.0                                                    617
      DO 523 K=1,KQ2
      C1=C1+EMR(I,K)*WR(K,J)+EME(I,K)*WC(K,J)
523   C2=C2+EME(I,K)*WB(K,J)-EMR(I,K)*WC(K,J)
      SR(I,J,NF)=C1
524   SE(I,J,NF)=C2
      GO TO 530
525   DO 528 I=1,KQ2
      C2=0.0
      DO 527 J=1,KQ2
      C1=0.0
      DO 526 K=1,KQ2
526   C1=C1+2.0*WB(I,K)*SR(K,J,1)-2.0*WC(I,K)*SE(K,J,1)
      C2=C2-C1*XZ(J)
527   DEVN(I,J)=C1+UR(I,J)
528   WCC(I,1)=C1
530   CONTINUE
```

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```
      CALL MATINV(DEVN,KQ2,WCC,1,DVN,ID)                                  694
      GO TO (540,531),ID
531   WRITE(6,133)                                                        696
      GO TC 599
540   DO 541 I=1,KQ2
      XR(I,1)=WCC(I,1)
      XE(I,1)=0.0
541   XZ1(I)=XZ1(I)+WCC(I,1)
      DO 543 I=1,KQ2
      C1=0.0
      C2=0.0
      DO 542 J=1,KQ2
      C1=C1+2.0*SR(I,J,1)*XZ1(J)
542   C2=C2+2.0*SE(I,J,1)*XZ1(J)
      XR(I,2)=C1
543   XE(I,2)=C2
      WRITE(6,134)                                                        726
      WRITE(6,139)                                                        727
      NH2=NH1
      DO 553 IK=1,NH1
      NF=IK-1
      HN=NF
      FRQ=HN*SPD*SFR1
      LALM=0
      IF(NF-1) 550,550,545
545   DO 549 I=1,KQ2
      C1=0.0
      C2=0.0
      DO 546 K=1,KQ2
      C1=C1+SR(I,K,NF)*XR(K,NF)-SE(I,K,NF)*XE(K,NF)
546   C2=C2+SR(I,K,NF)*XE(K,NF)+SE(I,K,NF)*XR(K,NF)
      XR(I,IK)=C1
      IF(ABS(C1)-AMLM)  547,547,549
547   IF(ABS(C2)-AMLM)  548,548,549
548   LALM=LALM+1
549   CONTINUE
      IF(LALM-4)  550,544,544
544   NH2=IK
550   WRITE(6,14)NF,FRQ,XR(1,IK),XE(1,IK),XR(2,IK),XE(2,IK),XR(3,IK),
     1XE(3,IK),XR(4,IK),XE(4,IK)                                          759
      DO 552 J=1,NS
      C1=0.0
      C2=0.0
      C3=0.0
      C4=0.0
      DO 551 K=1,KQ2
      C1=C1+XCS(K,J,IK)*XR(K,IK)-XSS(K,J,IK)*XE(K,IK)
      C2=C2+XCS(K,J,IK)*XE(K,IK)+XSS(K,J,IK)*XR(K,IK)
      C3=C3+YCS(K,J,IK)*XR(K,IK)-YSS(K,J,IK)*XE(K,IK)
551   C4=C4+YCS(K,J,IK)*XE(K,IK)+YSS(K,J,IK)*XR(K,IK)
      XCS(1,J,IK)=C1
      XSS(1,J,IK)=C2
      YCS(1,J,IK)=C3
552   YSS(1,J,IK)=C4
      IF(LALM-4)  553,537,559
553   CONTINUE
```

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```
539 DO 570 J=1,NS
    WRITE(6,131)J                                                          803
    WRITE(3,135)                                                           804
    DO 554 IK=1,NH2
    NF=IK-1
    HN=NF
    FRQ=HN*SPD*SFR1
554 WRITE(6,13)NF,FRQ,XCS(1,J,IK),XSS(1,J,IK),YCS(1,J,IK),YSS(1,J,IK)       808
    WRITE(6,133)                                                           813
    WRITE(6,137)                                                           814
    C1=WTST
555 C2=0.01745329*C1*SFR
    C3=XCS(1,J,1)
    C4=YCS(1,J,1)
    C5=0.0
    DO 556 IK=1,NH2
    C5=C5+C2
    C6=COS(C5)                                                             820
    C7=SIN(C5)                                                             821
    C3=C3+XCS(1,J,IK)*C6-XSS(1,J,IK)*C7
556 C4=C4+YCS(1,J,IK)*C6-YSS(1,J,IK)*C7
    C5=SQRT(C3*C3+C4*C4)                                                   828
    D1=C3
    D2=C4
    IF(C3) 564,557,561
557 IF(C4) 559,560,558
558 C6=90.0
    GO TO 567
559 C6=270.0
    GO TO 567
560 C6=0.0
    GO TO 567
561 IF(C4) 563,562,562
562 C7=0.0
    C8=57.295780
    GO TO 566
563 C7=360.0
    C8=-57.295780
    C4=-C4
    GO TO 566
564 C3=-C3
    C7=180.0
    C8=-57.295780
    IF(C4) 565,566,566
565 C8=-C8
    C4=-C4
566 C6=C7+C8*ATAN(C4/C3)                                                   848
567 WRITE(6,102)C1,D1,D2,C5,C6                                             849
    C1=C1+WTIN
    IF(WTFN+0.00001-C1) 570,570,555
570 CONTINUE
599 SPD=SPD+SPIN                                                           734
    IF(SPFN+0.00001-SPD)  600,600,260                                      735
600 NSP1=NSP1+1                                                            736
    IF(NSP-NSP1)  601,230,230                                              737
601 IF(INP)  602,190,602                                                   738
```

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```
.22 FORMAT( P5. 4.5)                                                    743
.27 FORMAT(1J, P. 5.5, .P7E.4.6)                                        749
 22 FORMAT(///.2H,HARMONIC NO.,13,.2H,FREQUENCY=,1PE.3.5,8H RAD/SEC)    767
 23 FORMAT(..H.BEARING COEFFICIENTS)                                    768
 24 FORMAT(5H STATION.X3HKXX9X7IIFRQ.BXX9X3HKXY9X7HFRQ.BXY9X3HKYY9X7HFR 769·
   .Q.BYX9X3HKYY9X7HFRQ.BYY)                                           770
 25 FORMAT(.H.ROTOR-BEARING IMPEDANCE MATRIX)                          771
 27 FORMAT( OH.REAL PART)
 28 FORMAT( 6H.IMAGINARY PART)
 30 FORMAT(.4H.REAL PART OF A-MATRIX IS SINGULAR)
.3. FORMAT( OH HARMONIC=,13,.2H  FREQUENCY=,1PE13.6)
.32 FORMAT( 3H ROTOR STATION NO.,12)
.33 FORMAT(.5H.FINAL MATRIX IS SINGULAR)
 34 FORMAT(49H,AMPLITUDES AT ROTOR STATION WITH MAGNETIC FORCES)
.35 FORMAT(.OH HARMONIC  FREQUENCY7X4HX(C).3X4HX(S)13X4HY(C)10X4HY(S))
.36 FORMAT(/// 2HCWHIRL ORBIT)
.37 FORMAT( 5H SHAFT ROTAT.DEG5X.HX.3X.HY9X9HAMPLITUDE4X.2HANGLE X-AMP
   .L)
.38 FORMAT(.P4. 4.5)
.39 FORMAT(..H HARMONIC   FREQUENCY5X4HX(C)8X4HX(S)8X4HY(C)8X4HY(S)6X8
   .HDX/DZ(C)+X6HDX/DZ(S)4X6HHDY/DZ(C)4X6HDY/DZ(S))
.40 FORMAT(4.H.REAL PART OF IMPEDANCE MATRIX IS SINGULAR)
.4. FORMAT(15, PE.6.4, P5E.2.4)
142 FORMAT(.7H.REAL PART OF S-MATRIX IS SINGULAR,K=,13)
143 FORMAT(.3H.INVERSE IMPEDANCE MATRIX)
.44 FORMAT(5+H.INFLUENCE COEFFICIENT MATRIX E (X(J)=E.E) FOR STATION.I
   .3)
.45 FORMAT(.6H.INFLUENCE MATRIX A (X(J)=A.X(KB)) FOR STATION,I3)
.46 FORMAT(.5H.INVERSION MATRIX FOR A IS SINGULAR)
.47 FORMAT(.5H.INVERSION MATRIX FOR E IS SINGULAR)
.48 FORMAT(.8H.INVERSION MATRIX FOR S IS SINGULAR,K=I3)
280    RETURN .
230    RETURN 2
.90    RETURN .
602    RETURN
       END                                                             776
```

```
      SUBROUTINE BB2
      COMMON/A/ BKXX(10,11),BCXX(10,11),BKXY(10,11),BCXY(10,11),
     1BKYX(10,11),BCYX(10,11),BKYY(10,11),BCYY(10,11),PHIN( 0,5),
     2PADM(10,5),PADK(10,5),PANG(10,5),DEVN(4,4),WCC(4,4),CRR(4,8),
     3CME(4,8),AMR(4,8),AME(4,8),WR(4,4),WE(4,4),WA(4,4),WB(4,4),
     4WC(4,4),WQ(4,4),WSQ(4,4),UR(4,4),UE(4,4),EMR(4,4),EMF(4,4),
     5XR(4,11),XE(4,11)
      COMMON/B/ PKXX(10,5,11),PCXX(10,5,11),PKKY(10,5,11),PCXY(10,5,11),
     1PKYX(10,5,11),PCYX(10,5,11),PKYY(10,5,11),PCYY(10,5,11),
     2GR(2,8,50),GE(2,8,50),SR(4,4,11),SE(4,4,11),XCS(4,50,11),
     3XSS(4,50,11),YCS(4,50,11),YSS(4,50,11)
      COMMON/C/ RM(30),RIP(30),RIT(30),RS(30),RW(30),RO(30),RL(30),
     1DVXA(30),DVXB(30),DVXC(30),DVXD(30),DVYA(30),DVYB(30),DVYC(30),
     2DVYD(30),DVUX(30),DVUY(30),DMUX(30),DMUY(30),DMXA(30),B1(30),
     3B2(30),B3(30),B4(30),B5(30),B6(30),B7(30),B8(30),B9(30),B10(30),
     4PMX(10),PKX(10),PDX(10),PHY(10),PKY(10),PDY(10),SXX(10),DXX(10),
     5SXY(10),DXY(10),SYX(10),DYX(10),SYY(10),DYY(10),LB(10),
     6XZ(4),XZ1(4)
      COMMON/D/A1,A2,A3,A4,A5,A6,A7,A8,NF,FRQ,KC,SCF1,NDIA,KQ2,NS,KB,
     *  IK,NH2,HN,AMLM,SPD,SFR1,SFR,WTST,WTIN,WTF, NH1
      COMMON/E/KQ1,KQ3,C1,C2,NB,KA,NRP,NPD,INC,NH,NSP,INP,YP,DNST,SHM,
     *SPST,SPFN,SPIN,SCF,QZ,QZP,K1,K2, WTFN
      COMMON/F/BMXC,BMXS,BMYC,BMYS,VXC,VXS,VYC,VYS,XC,XS,YC,YS,DXC,DXS,
     *DYS,C3,C4, NS1, NPD1, NPD2, DYC, NSP1
      DO 485 I=1,KQ3
      BMXC=0.0                                                           383
      BMXS=0.0                                                           384
      BMYC=0.0                                                           385
      BMYS=0.0                                                           386
      VXC=0.0                                                            387
      VXS=0.0                                                            388
      VYC=0.0                                                            389
      VYS=0.0                                                            390
      XC=0.0                                                             391
      XS=0.0                                                             392
      YC=0.0                                                             393
      YS=0.0                                                             394
      DXC=0.0                                                            395
      DXS=0.0                                                            396
      DYC=0.0                                                            397
      DYS=0.0                                                            398
      DVUX(KB)=0.0
      DVUY(KB)=0.0
      DMUX(KB)=0.0
      DMUY(KB)=0.0
      GO TO(461,462,463,464,465,469,468,472),I
461   XC=0.00                                                            400
      GO TO 475                                                          401
462   YC=0.00                                                            402
      GO TO 475                                                          403
463   DXC=0.00                                                           404
      GU TO 475                                                          405
464   DYC=0.0                                                            406
      GU TO 475                                                          407
465   IF(KC)  467,465,466
```

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```
465 IF(KA)  463,467,467
467 DVUX(KB)=...
    GO TO 475
468 DMUX(KB)=...
    GO TO 475
469 IF(KC)  47.,47.,47.
470 IF(KA)  47.,47.,47.
471 DVUY(KB)=...
    GO TO 475
472 DMUY(KB)=...
475 DO 480 J=_,NS                                                            421
    C_=DMXA(J)                                                               422
    C2=FRQ*SPD*RIP(J)                                                        423
    GR(_,1,J)=XC
    GR(2,1,J)=YC
    GE(_,1,J)=XS
    GE(2,1,J)=YS
    IF(J-KB) 477,476,477                                                     424
476 CMR(_,1)=XC                                                              425
    CME(_,1)=XS                                                              426
    CMR(2,1)=YC                                                              427
    CME(2,1)=YS                                                              428
    CMR(3,1)=DXC                                                             429
    CME(3,1)=DXS                                                             430
    CMR(4,1)=DYC                                                             431
    CME(4,1)=DYS                                                             432
477 A_=BMXC-C_*DXC-C_*DYS-DMUX(J)                                            433
    A2=BMXS-C_*DXS+C_*DYC                                                    434
    A3=BMYC-C_*DYC+C_*DXS-DMUY(J)                                            435
    A4=BMYS-C_*DYS-C2*DXC                                                    436
    A5=VXC+DVXA(J)*XC+DVXB(J)*XS-DVXC(J)*YC+DVXD(J)*YS+DVUX(J)               437
    A6=VXS-DVXB(J)*XC+DVXA(J)*XS-DVXD(J)*YC-DVXC(J)*YS                       438
    A7=VYC-DVYC(J)*XC+DVYD(J)*XS+DVYA(J)*YC+DVYB(J)*YS+DVUY(J)               439
    A8=VYS-DVYD(J)*XC-DVYC(J)*XS-DVYB(J)*YC+DVYA(J)*YS                       440
    IF(NS-J)  480,480,478                                                    441
478 C_=XC                                                                    442
    C2=XS                                                                    443
    C3=YC                                                                    444
    C4=YS                                                                    445
    BMXC=C_*B9(J)+DXC*B_C(J)+A_*B2(J)+A5*B3(J)                               446
    BMXS=C2*B9(J)+DXS*B_C(J)+A2*B2(J)+A6*B3(J)                               447
    BMYC=C3*B9(J)+DYC*B_C(J)+A3*B2(J)+A7*B3(J)                               448
    BMYS=C4*B9(J)+DYS*B_C(J)+A4*B2(J)+A6*B3(J)                               449
    VXC=C_*B8(J)+DXC*B9(J)+A_*B3(J)+A5*B_(J)                                 450
    VXS=C2*B8(J)+DXS*B9(J)+A2*B3(J)+A6*B_(J)                                 451
    VYC=C3*B8(J)+DYC*B9(J)+A3*B3(J)+A7*B_(J)                                 452
    VYS=C4*B8(J)+DYS*B9(J)+A4*B3(J)+A8*B_(J)                                 453
    XC=C_*B_(J)+DXC*B3(J)+A_*B4(J)+A5*B7(J)                                  454
    XS=C2*B_(J)+DXS*B3(J)+A2*B4(J)+A6*B7(J)                                  455
    YC=C3*B_(J)+DYC*B3(J)+A3*B4(J)+A7*B7(J)                                  456
    YS=C4*B_(J)+DYS*B3(J)+A4*B4(J)+A8*B7(J)                                  457
    DXC=C_*B5(J)+DXC*B_(J)+A_*B2(J)+A5*B4(J)                                 458
    DXS=C2*B5(J)+DXS*B_(J)+A2*B2(J)+A6*B4(J)                                 459
    DYC=C3*B5(J)+DYC*B_(J)+A3*B2(J)+A7*B4(J)                                 460
    DYS=C4*B5(J)+DYS*B_(J)+A4*B2(J)+A6*B4(J)                                 461
480 CONTINUE                                                                 462
```

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```
      AMR(1,I)=A1                                            463
      AME(1,I)=A2                                            464
      AMR(2,I)=A3                                            465
      AME(2,I)=A4                                            466
      AMR(3,I)=A5                                            467
      AME(3,I)=A6                                            468
      AMR(4,I)=A7                                            469
      AME(4,I)=A8                                            470
  485 CONTINUE                                               471
      RETURN
      END
```

NII-4077

```
C      MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS    MAT1    1
C      NOVEMBER 1962   S GOOD   DAVID TAYLOR MODEL BASIN    AM MAT.       MAT1    2
C                                                                        MAT1    4
       SUBROUTINE MATINV(A,N,B,M,DETERM,ID)                              MAT1    5
C                                                                        MAT1    6
C      GENERAL FORM OF DIMENSION STATEMENT                               MAT1    7
C                                                                        MAT1    8
       DIMENSION A(4,4),B(4,1)
       DIMENSION INDEX(4,3)
       EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)         MAT1   11
C                                                                        MAT1   12
C      INITIALIZATION                                                    MAT1   13
C                                                                        MAT1   14
       M=M.                                                              MAT1   15
       N=N.                                                              MAT1   16
       DO 8 I=1,N
       K1=1
       K2=1
       DO 6 J=1,N
       IF(A(I,J)) 3,4,3
   3   K1=0
   4   IF(A(J,I)) 5,6,5
   5   K2=0
   6   CONTINUE
       IF(K1+K2) 8,8,7
   7   ID=2
       DETERM=0.0
       GO TO 740
   8   CONTINUE
  10   DETERM=1.0
  15   DO 20 J=1,N                                                       MAT1   18
  20   INDEX(J,3) = 0                                                    MAT1   19
  30   DO 550 I=1,N                                                      MAT1   20
C                                                                        MAT1   21
C      SEARCH FOR PIVOT ELEMENT                                          MAT1   22
C                                                                        MAT1   23
  40   AMAX=0.0                                                          MAT1   24
  45   DO 105 J=1,N                                                      MAT1   25
       IF(INDEX(J,3)-1) 60, 105, 60                                      MAT1   26
  60   DO 100 K=1,N                                                      MAT1   27
       IF(INDEX(K,3)-1) 80, 100, 715                                    MAT1   28
  80   IF(AMAX-ABS(A(J,K))) 85,100,100                                   MAT1   29
  85   IROW=J                                                            MAT1   30
  90   ICOLUM=K                                                          MAT1   31
       AMAX=ABS(A(J,K))                                                  MAT1   32
 100   CONTINUE                                                          MAT1   33
 105   CONTINUE                                                          MAT1   34
       INDEX(ICOLUM,3) = INDEX(ICOLUM,3) +1                             MAT1   35
 260   INDEX(I,1)=IROW                                                   MAT1   36
 270   INDEX(I,2)=ICOLUM                                                 MAT1   37
C                                                                        MAT1   38
C      INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL                 MAT1   39
C                                                                        MAT1   40
 130   IF (IROW-ICOLUM) 140, 310, 140                                    MAT1   41
 140   DETERM=-DETERM                                                    MAT1   42
```

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```
150 DO 200 L=1,M                                    MAT1 43
160 SWAP=A(IROW,L)                                  MAT1 44
170 A(IROW,L)=A(ICOLUM,L)                           MAT1 45
200 A(ICOLUM,L)=SWAP                                MAT1 46
    IF(M) 3.0, 3.0, 2.0                             MAT1 47
210 DO 250 L=1,M                                    MAT1 48
220 SWAP=B(IROW,L)                                  MAT1 49
230 B(IROW,L)=B(ICOLUM,L)                           MAT1 50
250 B(ICOLUM,L)=SWAP                                MAT1 51
C                                                   MAT1 52
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT             MAT1 53
C                                                   MAT1 54
310 PIVOT   =A(ICOLUM,ICOLUM)                       MAT1 55
    DETERM=DETERM*PIVOT                             MAT1 56
330 A(ICOLUM,ICOLUM)=1.0                            MAT1 57
340 DO 350 L=1,N                                    MAT1 58
350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT                   MAT1 59
355 IF(M) 380, 380, 360                             MAT1 60
360 DO 370 L=1,M                                    MAT1 61
370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT                   MAT1 62
C                                                   MAT1 63
C     REDUCE NON-PIVOT ROWS                         PAT1 64
C                                                   MAT1 65
380 DO 550 L1=1,N                                   MAT1 66
390 IF(L1-ICOLUM) 400, 550, 400                     MAT1 67
400 T=A(L1,ICOLUM)                                  MAT1 68
420 A(L1,ICOLUM)=0.0                                MAT1 69
430 DO 450 L=1,N                                    MAT1 70
450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T                   MAT1 71
455 IF(M) 550, 550, 460                             MAT1 72
460 DO 500 L=1,M                                    MAT1 73
500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T                   MAT1 74
550 CONTINUE                                        MAT1 75
C                                                   MAT1 76
C     INTERCHANGE COLUMNS                           MAT1 77
C                                                   MAT1 78
600 DO 710 I=1,N                                    MAT1 79
610 L=N+1-I                                         MAT1 80
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630        MAT1 81
630 JROW=INDEX(L,1)                                 MAT1 82
640 JCOLUM=INDEX(L,2)                               MAT1 83
650 DO 705 K=1,N                                    MAT1 84
660 SWAP=A(K,JROW)                                  MAT1 85
670 A(K,JROW)=A(K,JCOLUM)                           MAT1 86
700 A(K,JCOLUM)=SWAP                                MAT1 87
705 CONTINUE                                        MAT1 88
710 CONTINUE                                        MAT1 89
    DO 730 K = 1,N                                  MAT1 90
    IF(INDEX(K,3) -1) 715,720,715                   MAT1 91
720 CONTINUE                                        MAT1 94
730 CONTINUE                                        MAT1 95
    ID=1                                            MAT1 96
C   LAST CARD OF PROGRAM                            MAT1 96
740 RETURN                                          MAT1 97
715 ID =2                                           MAT1 92
    GO TO 740                                       MAT1 93
```

```
   11      2      7     -1      0      4      1      5      2      0      1
 30000000.0      0.283          8630000.0
   6.15    62.4    31.0     1.3     3.5     3.5    3.339
   1.61     0.0     0.0     1.45    3.5     3.6    3.2
   0.0      0.0     0.0     2.37    3.5     3.65   3.062
   0.0      0.0     0.0     4.83    3.5     3.65   3.062
   0.0      0.0     0.0     1.67    2.775   3.25   2.0
   3.1     12.0    12.0     2.1     3.5     3.7    1.04
   0.0      0.0     0.0     2.1     3.5     3.7    1.04
   3.1     12.0    12.0     1.67    2.775   3.25   2.0
   0.0      0.0     0.0     3.49    3.5     3.65   3.062
   0.0      0.0     0.0     4.39    3.5     3.65   3.062
   9.21    56.0    28.0     0.0     1.0     0.0    0.0
   4     10
   0.000000E+00    3.400000E+02    2.000000E+01    1.000000E-20
   8000.0          21971.0         4657.0          2.0            1.0
  20000.0          88200.0
   0.0             0.0             42000.0          0.0
   0.0             0.0              0.0           -42000.0
  42000.0          0.0              0.0             0.0
   0.0           -42000.0           0.0             0.0
   0.0             0.0              0.0           -42000.0
   0.0             0.0           -42000.0            0.0
   0.0           -42000.0           0.0             0.0
 -42000.0          0.0              0.0             0.0
   0.0             0.0              0.002           0.0
   0.13            0.57            300000.0         45.0
 39600.0   25560.0   19550.0  -19700.0    2455.0    -605.0    2720.0    1202.0
 63000.0    7360.0    1470.0   -4810.0    2020.0   -1023.0    3710.0    1377.0
 68300.0    2340.0   -1227.0   -1202.0     268.5    -677.0    5875.0     843.0
 70300.0    1190.0   -1650.0    -575.0    -677.0    -415.0    7060.0     524.0
 71200.0     742.0   -1828.0    -352.0   -1202.0    -281.0    7700.0     352.0
 71900.0     537.0   -1937.0    -241.5   -1558.0    -203.0    8130.0     261.0
   0.13            0.57            400000.0        135.0
162500.0   86200.0   66500.0  -94100.0    9480.0    2440.0   14350.0   -1970.0
229500.0    7480.0   -4270.0   -6810.0    7610.0   -3910.0   17770.0    4370.0
226500.0    1435.0     735.0   -1705.0    2970.0   -1370.0   22500.0    1203.0
224500.0     534.0     969.0    -869.0    1435.0    -745.0   23400.0     607.0
224000.0     274.0     434.0    -544.0     701.0    -484.0   23800.0     378.0
223500.0     160.5       0.0    -384.0     277.0    -348.0   24050.0     261.0
   0.13            0.57            300000.0         45.0
 39600.0   25560.0   19550.0  -19700.0    2455.0    -605.0    2720.0    1202.0
 63000.0    7360.0    1470.0   -4810.0    2020.0   -1023.0    3710.0    1377.0
 68300.0    2340.0   -1227.0   -1202.0     268.5    -677.0    5875.0     843.0
 70300.0    1190.0   -1650.0    -575.0    -677.0    -415.0    7060.0     524.0
 71200.0     742.0   -1828.0    -352.0   -1202.0    -281.0    7700.0     352.0
 71900.0     537.0   -1937.0    -241.5   -1558.0    -203.0    8130.0     261.0
   0.13            0.57            400000.0        135.0
162500.0   86200.0   66500.0  -94100.0    9480.0    2440.0   14350.0   -1970.0
229500.0    7480.0   -4270.0   -6810.0    7610.0   -3910.0   17770.0    4370.0
226500.0    1435.0     735.0   -1705.0    2970.0   -1370.0   22500.0    1203.0
224500.0     534.0     969.0    -869.0    1435.0    -745.0   23400.0     607.0
224000.0     274.0     434.0    -544.0     701.0    -484.0   23800.0     378.0
223500.0     160.5       0.0    -384.0     277.0    -348.0   24050.0     261.0
  7757.0          9557.00         300.0           2.0            1.0
 20000.0          88200.0
   0.0             0.0             42000.0          0.0
   0.0             0.0              0.0           -42000.0
  42000.0          0.0              0.0             0.0
   0.0           -42000.0           0.0             0.0
```

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```
  0.0          0.0          0.0        -42000.0
  0.0          0.0        -42000.0        0.0
  0.0        -42000.0        0.0          0.0
-42000.0       0.0          0.0          0.0
  0.0          0.0          0.002        0.0
  0.13         0.57       300000.0       45.0
 39600.0     25560.0     19550.0    -19700.0    2455.0     -605.0     2720.0     1202.0
 63000.0      7360.0      1470.0     -4810.0    2020.0    -1023.0     3710.0     1377.0
 68300.0      2340.0     -1227.0     -1202.0     268.5     -677.0     5875.0      843.0
 70300.0      1190.0     -1650.0      -575.0    -677.0     -415.0     7060.0      524.0
 71200.0       742.0     -1828.0      -352.0   -1202.0     -281.0     7700.0      352.0
 71900.0       537.0     -1937.0      -241.5   -1558.0     -203.0     8130.0      261.0
  0.13         0.57       400000.0      135.0
162500.0     86200.0     66500.0    -94100.0    9480.0     2440.0    14350.0    -1970.0
229500.0      7480.0     -4270.0     -6810.0    7610.0    -3910.0    17770.0     4370.0
226500.0      1435.0       735.0     -1705.0    2970.0    -1370.0    22500.0     1203.0
224500.0       534.0       969.0      -869.0    1435.0     -745.0    23400.0      607.0
224000.0       274.0       434.0      -544.0     701.0     -484.0    23800.0      378.0
223500.0       160.5         0.0      -384.0     277.0     -348.0    24050.0      261.0
  0.13         0.57       300000.0       45.0
 39600.0     25560.0     19550.0    -19700.0    2455.0     -605.0     2720.0     1202.0
 63000.0      7360.0      1470.0     -4810.0    2020.0    -1023.0     3710.0     1377.0
 68300.0      2340.0     -1227.0     -1202.0     268.5     -677.0     5875.0      843.0
 70300.0      1190.0     -1650.0      -575.0    -677.0     -415.0     7060.0      524.0
 71200.0       742.0     -1828.0      -352.0   -1202.0     -281.0     7700.0      352.0
 71900.0       537.0     -1937.0      -241.5   -1558.0     -203.0     8130.0      261.0
  0.13         0.57       400000.0      135.0
162500.0     86200.0     66500.0    -94100.0    9480.0     2440.0    14350.0    -1970.0
229500.0      7480.0     -4270.0     -6810.0    7610.0    -3910.0    17770.0     4370.0
226500.0      1435.0       735.0     -1705.0    2970.0    -1370.0    22500.0     1203.0
224500.0       534.0       969.0      -869.0    1435.0     -745.0    23400.0      607.0
224000.0       274.0       434.0      -544.0     701.0     -484.0    23800.0      378.0
223500.0       160.5         0.0      -384.0     277.0     -348.0    24050.0      261.0
```

TURBINE DRIVEN 4 POLE ALTERNATOR RESPONSE 4-21-1967

STATIONS	BEARINGS	MAGN.ST	F+M/F/M	RIG.PED	NO.PADS	COMPRESS	HARMONICS	NO.SPEEDS	DIAGNOS	INPUT
11	2	7	-1	0	4	1	5	2	0	1

YOUNGS MOD.   DENSITY   (SHAPE FACT)*G
3.000000E 07 2.830300E-01 8.630000E 06

ROTOR DATA

STATION	MASS,LBS	PULAR MOM.IN.	TRANSV.MOM.IN	LENGTH	OUT.DIA(STIFF)	OUT.DIA(MASS)	INNER DIA.
1	6.150000E 00	6.240000E 01	3.100000E 01	1.300000E 00	3.500000E 00	3.500000E 00	3.339000E 00
2	1.610000E 00	0.	0.	1.450000E 00	3.500000E 00	3.600000E 00	3.200000E 00
3	0.	0.	0.	2.370000E 00	3.500000E 00	3.650000E 00	3.062000E 00
4	0.	0.	0.	4.830000E 00	3.500000E 00	3.650000E 00	3.062000E 00
5	0.	0.	0.	1.670000E 00	2.775000E 00	3.250000E 00	2.000000E 00
6	3.100000E 00	1.200000E 01	1.200000E 01	2.100000E 00	3.500000E 00	3.700000E 00	4.040000E 00
7	0.	0.	0.	2.100000E 00	3.500000E 00	3.700000E 00	1.040000E 00
8	3.100000E 00	1.200000E 01	1.200000E 01	1.670000E 00	2.775000E 00	3.250000E 00	2.000000E 00
9	0.	0.	0.	3.490000E 00	3.500000E 00	3.650000E 00	3.062000E 00
10	0.	0.	0.	4.390000E 00	3.500000E 00	3.650000E 00	3.062000E 00
11	9.210000E 00	5.600000E 01	2.800000E 01	0.	1.000000E 00	0.	0.

BEARING STATIONS
4 10

MAGNETIC FORCE DATA

Q(0).FORCE   Q(0).MOMENT
2.000000E 04 8.820000E 04

MATRIX OF COSINE COMPONENTS

0.	0.	4.200000E 04	0.
0.	0.	0.	-4.200000E 04
4.200000E 04	0.	0.	0.
0.	-4.200000E 04	0.	0.

MATRIX OF SINE COMPONENTS

0.	0.	0.	-4.200000E 04
0.	0.	-4.200000E 04	0.
0.	-4.200000E 04	0.	0.
-4.200000E 04	0.	0.	0.

COORDINATES OF STATIC ROTOR ECCENTRICITY
0.   0.   2.000000E-03   0.

BEARING DATA

BEARING AT STATION 4

PAD NO. 1
PITCH MOM.IN.   PAD MASS   PIVOT STIFFN.   PIVOT ANGLE
1.300000E-01   5.700000E-01   3.000000E 05   4.500000E-01

HARMONIC	KXX	W*BXX	KXY	W*BXY	KYX	W*BYX	KYY	W*BYY
0	3.960000E 04	2.556000E 04	1.955000E 04	-1.970000E 04	2.455000E 03	-6.050000E 02	2.720000E 03	1.202000E 03
1	6.300000E 04	7.360000E 03	1.470000E 03	-4.810000E 03	2.020000E 03	-1.023000E 03	3.710000E 03	1.377000E 03
2	4.850000E 04	2.340000E 03	-1.227000E 03	-1.202000E 03	2.485000E 02	-6.770000E 02	5.075000E 02	8.430000E 02
3	7.030000E 04	1.190000E 03	-1.650000E 03	-5.750000E 03	-6.770000E 02	-4.150000E 02	7.060000E 02	5.240000E 02
4	7.120000E 04	7.420000E 02	-1.828000E 02	-3.520000E 03	-1.202000E 02	-2.810000E 02	7.700000E 02	3.520000E 02
5	7.190000E 04	5.370000E 02	-1.937000E 03	-2.415000E 02	-1.558000E 02	-2.030000E 02	8.130000E 02	2.610000E 02

PAD NO. 2
PITCH MOM.IN.   PAD MASS   PIVOT STIFFN.   PIVOT ANGLE
1.300000E-01   5.700000E-01   4.000000E 05   1.350000E 02

HARMONIC	KXX	W*BXX	KXY	W*BXY	KYX	W*BYX	KYY	W*BYY
0	1.625000E 05	8.620000E 04	6.650000E 04	-9.410000E 04	9.680000E 03	2.440000E 03	1.435000E 04	-1.970000E 03
1	2.295000E 05	7.480000E 03	-4.270000E 03	-6.810000E 03	7.610000E 03	-3.910000E 03	1.777000E 03	4.370000E 03
2	2.265000E 05	1.435000E 03	7.350000E 03	-1.705000E 02	2.970000E 03	-1.370000E 02	2.250000E 03	1.203000E 03
3	2.245000E 05	5.340000E 02	5.690000E 02	-8.690000E 02	1.435000E 03	-7.330000E 02	2.340000E 03	6.070000E 03
4	2.240000E 05	2.740000E 02	4.300000E 02	-5.440000E 02	7.010000E 02	-4.840000E 02	2.380000E 02	3.780000E 02
5	2.235000E 05	1.605000E 02	0.	-3.480000E 02	2.770000E 02	-3.460000E 02	2.405000E 02	2.610000E 02

BEARING AT STATION 10

PAD NO. 1
PITCH MOM.IN.   PAD MASS   PIVOT STIFFN.   PIVOT ANGLE
1.300000E-01   5.700000E-01   3.000000E 05   4.500000E-01

HARMONIC	KXX	W*BXX	KXY	W*BXY	KYX	W*BYX	KYY	W*BYY
0	3.960000E 04	2.556000E 04	1.955000E 04	-1.970000E 04	2.455000E 03	-6.050000E 02	2.720000E 03	1.202000E 03
1	6.300000E 04	7.360000E 03	1.470000E 03	-4.810000E 03	2.020000E 03	-1.023000E 03	3.710000E 03	1.377000E 03
2	6.830000E 04	2.340000E 03	-1.227000E 03	-1.202000E 03	2.485000E 02	-6.770000E 02	5.075000E 02	6.430000E 02
3	7.030000E 04	1.190000E 03	-1.650000E 03	-5.750000E 03	-6.170000E 02	-4.150000E 02	7.060000E 02	5.240000E 02
4	7.120000E 04	7.420000E 02	-1.828000E 02	-3.520000E 03	-1.202000E 02	-2.810000E 02	7.700000E 02	3.520000E 02

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5   7.190000E 04   5.370000E 02 -1.937000E 03 -2.415000E 02 -1.558000E 03 -2.030000E 02   8.130000E 03   2.610000E 02

PAD NO. 2
PITCH MOM.IN.   PAD MASS   PIVOT STIFFN.   PIVOT ANGLE
1.300000E-01   5.700000E-01   4.000000E 05   1.350000E 02

HARMONIC	KXX	M*BXX	KXY	M*BXY	KYX	M*BYX	KYY	M*BYY
0	1.625000E 05	8.620000E 04	6.650000E 04	-9.410000E 04	9.480000E 03	2.440000E 03	1.435000E 04	-1.970000E 03
1	2.295000E 05	7.480000E 03	-4.270000E 03	-6.810000E 03	7.610000E 03	-3.910000E 03	1.777000E 04	4.370000E 03
2	2.265000E 05	1.435000E 03	7.350000E 02	-1.705000E 03	2.970000E 03	-1.370000E 03	2.250000E 04	1.200000E 03
3	2.245000E 05	5.340000E 02	9.690000E 02	-8.690000E 02	1.435000E 03	-7.450000E 02	2.340000E 04	6.070000E 02
4	2.240000E 05	2.740000E 02	4.340000E 02	-5.440000E 02	7.010000E 02	-4.840000E 02	2.380000E 04	3.780000E 02
5	2.235000E 05	1.605000E 02	0.	-3.840000E 02	2.770000E 02	-3.480000E 02	2.405000E 04	2.610000E 02

ROTOR SPEED= 8.000000E 03 RPM

AMPLITUDES AT ROTOR STATION WITH MAGNETIC FORCES

HARMONIC	FREQUENCY	X(C)	X(S)	Y(C)	Y(S)	DX/DZ(C)	DX/DZ(S)	DY/DZ(C)	DY/DZ(S)
0	0.	-3.4861E-05	0.	4.4233E-06	0.	-4.1316E-06	0.	4.2108E-06	0.
1	1.6755E 03	1.0993E-03	1.1696E-03	1.1696E-03	-1.0993E-03	1.3287E-04	1.1032E-05	1.1032E-05	-1.329?E-04
2	3.3510E 03	-2.7853E-12	0.	4.2930E-12	0.	1.5810E-13	0.	-4.7885E-13	0.
3	5.0265E 03	4.4291E-15	0.	1.1720E-14	0.	8.0273E-16	0.	1.1315E-15	0.
4	6.7021E 03	5.5668E-18	0.	-6.4772E-18	0.	-2.0595E-18	0.	5.2307E-18	0.
5	8.3776E 03	-9.8293E-21	0.	-2.5198E-20	0.	-2.8520E-21	0.	-2.4617E-21	0.

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ROTOR STATION NO. 1

HARMONIC	FREQUENCY	X(C)	X(S)	Y(C)	Y(S)
0	0.	2.158623E-05	-0.	-4.207429E-05	-0.
1	1.675516E 03	-7.658826E-04	9.240380E-04	9.240351E-04	7.658811E-04
2	3.351032E 03	4.883062E-12	-7.188894E-12	-1.031933E-11	-3.611345E-12
3	5.026548E 03	1.389998E-15	-1.672908E-15	6.110268E-16	7.691916E-17
4	6.702065E 03	-1.769759E-18	1.304734E-17	4.763008E-18	6.314209E-18
5	8.377581E 03	-8.687957E-22	-3.164666E-21	7.643583E-21	6.913349E-22

WHIRL ORBIT

SHAFT ROTAT,DEG	X	Y	AMPLITUDE	ANGLE X-AMPL
0.	-7.442964E-04	8.819608E-04	1.154050E-03	1.301614E 02
2.000000E 01	-1.159074E-03	1.734788E-04	1.171984E-03	1.714877E 02
4.000000E 01	-1.021408E-03	-6.358629E-04	1.203160E-03	2.119037E 02
6.000000E 01	-3.957128E-04	-1.167364E-03	1.232610E-03	2.512744E 02
8.000000E 01	4.252409E-04	-1.172330E-03	1.247072E-03	2.899373E 02
1.000000E 02	1.057320E-03	-6.484364E-04	1.240321E-03	3.284799E 02
1.200000E 02	1.204768E-03	1.591807E-04	1.215238E-03	7.526646E 00
1.400000E 02	7.985917E-04	8.726284E-04	1.182890E-03	4.753660E 01
1.600000E 02	2.884613E-05	1.158076E-03	1.158436E-03	8.857313E 01
1.800000E 02	-7.442965E-04	8.819607E-04	1.154050E-03	1.301614E 02
2.000000E 02	-1.159074E-03	1.734786E-04	1.171984E-03	1.714877E 02
2.200000E 02	-1.021408E-03	-6.358630E-04	1.203160E-03	2.119038E 02
2.400000E 02	-3.957126E-04	-1.167364E-03	1.232610E-03	2.512744E 02
2.600000E 02	4.252411E-04	-1.172330E-03	1.247072E-03	2.899373E 02
2.800000E 02	1.057320E-03	-6.484362E-04	1.240321E-03	3.284799E 02
3.000000E 02	1.204768E-03	1.591810E-04	1.215238E-03	7.526657E 00
3.200000E 02	7.985916E-04	8.726285E-04	1.182890E-03	4.753661E 01

ROTOR STATION NO. 2

HARMONIC	FREQUENCY	X(C)	X(S)	Y(C)	Y(S)
0	0.	1.624258E-05	-0.	-3.789821E-05	-0.
1	1.675516E 03	-5.935989E-04	9.294029E-04	9.294031E-04	5.935975E-04
2	3.351032E 03	3.538685E-12	-6.459052E-12	-7.625767E-12	-3.244694E-12
3	5.026548E 03	1.238280E-15	-1.845368E-15	1.875511E-15	1.555076E-17
4	6.702065E 03	6.462117E-18	2.093138E-17	-1.120950E-17	1.070200E-17
5	8.377581E 03	-2.991946E-22	-1.151740E-20	1.193816E-20	7.676124E-22

WHIRL ORBIT

SHAFT ROTAT,DEG	X	Y	AMPLITUDE	ANGLE X-AMPL
0.	-5.773563E-04	8.915018E-04	1.062128E-03	1.229280E 02
2.000000E 01	-1.035889E-03	2.925064E-04	1.076395E-03	1.642318E 02
4.000000E 01	-1.002118E-03	-4.61089lE-04	1.103106E-03	2.047078E 02
6.000000E 01	-4.918444E-04	-1.016609E-03	1.129392E-03	2.441832E 02
8.000000E 01	2.561686E-04	-1.114271E-03	1.143338E-03	2.829472E 02
1.000000E 02	8.919176E-04	-7.082262E-04	1.138904E-03	3.215487E 02
1.200000E 02	1.117928E-03	1.147245E-05	1.117987E-03	5.879621E-01
1.400000E 02	8.284482E-04	7.080700E-04	1.089812E-03	4.052032E 01
1.600000E 02	1.589279E-04	1.055621E-03	1.067517E-03	8.143819E 01
1.800000E 02	-5.773564E-04	8.915017E-04	1.062128E-03	1.229280E 02
2.000000E 02	-1.035889E-03	2.925062E-04	1.076395E-03	1.642318E 02
2.200000E 02	-1.002118E-03	-4.610892E-04	1.103106E-03	2.047079E 02
2.400000E 02	-4.918442E-04	-1.016609E-03	1.129392E-03	2.441832E 02
2.600000E 02	2.561688E-04	-1.114271E-03	1.143338E-03	2.829472E 02
2.800000E 02	3.919177E-04	-7.082260E-04	1.138904E-03	3.215487E 02
3.000000E 02	1.117928E-03	1.147263E-05	1.117987E-03	5.879718E-01
3.200000E 02	8.284481E-04	7.080701E-04	1.089812E-03	4.052033E 01

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2. E. T. Whittaker and G. N. Watson, "Modern Analysis," Cambridge, New York, 1927.

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13. ABSTRACT

This volume presents an analytical investigation of the vibrations induced in an alternator rotor by the generated electromagnetic forces. Formulas are given from which the magnetic forces can be calculated for three brushless alternator types: (1) the homopolar generator, (2) the heteropolar inductor generator, and (3) the two-coil Lundell generator. Numerical examples are given to illustrate the practical use of the formulas.

Two computer programs have been written for evaluation of the effect of the magnetic forces on the rotor. Manuals are provided for both programs, containing listings of the programs and giving detailed instructions for preparation of input data and for performing the calculations. The first computer program examines the stability of the rotor and the second program calculates the amplitude response of the rotor resulting from a built-in eccentricity and misalignment between the axes of the rotor and the alternator stator. Sample calculations are provided.

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14. KEY WORDS		LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
Alternators							
Bearings							
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